



LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

CAPRICORN NORTH DISTRICT

GRADE 12

MATHEMATICS P1

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LEVEL 1, 2, 3 & 4 SOLUTIONS MANUAL

Mathematics algebra manual solution

Solutions for level 1

1.1	$x=0$ or $x=2$	2
1.2	$x = 0$ or $x = -6$	2
1.3	$x^2 - 3x - 16 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-16)}}{2(1)}$ $x = 5.77$ or $x = -2.77$	4
1.4	$x = 2$ or $x = -5$	2
1.5	$x^2 + 6x + 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-6 \pm \sqrt{6^2 - 4(1)(1)}}{2(1)}$ $x = -0.17$ or $x = -5.83$	4
1.6	$(x - 5)(x - 4) = 0$ $x = 5$ or $x = -4$	2
1.7	$x = 0$ or $x = 3$	2
2.1	$x^2 + 2x - 1 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)}$ $x = 0,41$ or $x = -2,41$	3
2.2	$2x^2 - 3x - 7 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$ $x = 2.77$ or $x = -1.27$	3
2.3	$3x^2 + 8x + 2 = 0$ $x = \frac{-8 \pm \sqrt{8^2 - 4(3)(2)}}{2(3)}$ $x = -0,28$ or $x = -2,39$	4
2.4	$3x^2 - 4x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$ $x = -1.72$ or $x = -0.39$	4
2.5	$x^2 - 9 = 0$	2

	$(x - 3)(x + 3) = 0$ $x = \pm 3$	
2.6	$3^{3x+1} = 9^{2x-4}$ $3^{3x+1} = 3^{2(2x-4)}$ $3x + 1 = 4x - 8$ $x = 9$	3
2.7	$x - \frac{3}{x} = -2$ $x^2 + 2x - 3 = 0$ $(x + 3)(x - 1) = 0$ $x = -3 \text{ or } x = 1$	4
2.8	$x - 5 + \frac{2}{x} = 0$ $x^2 - 5x + 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(2)}$ $x = 4.56 \text{ or } x = 0.44$	4
2.9	$x + 2 = \frac{2}{x + 1}$ $(x + 2)(x + 1) = 2$ $x^2 + 3x + 2 - 2 = 0$ $x(x + 3) = 0$ $x = 0 \text{ or } x = -3$	4
3.1	$4 - p \geq 0$ $-p \geq -4$ $p \leq 4$	2
3.2		
3.2.1	$x = \frac{-5 \pm \sqrt{20 - 8k}}{6}$ $k = -2$ $k = 2$	2
3.2.2	$k = -3$	1
3.3	$2x^2 - 3x - p = 0$ $2(2)^2 - 3(2) - p = 0$ $p = 2$ $2x^2 - 3x - 2 = 0$ $(2x + 1)(x - 2) = 0$ $x = -\frac{1}{2} \text{ or } x = 2$	4

Solution for level 2(q2)

4.1	$2 - x = \sqrt{2 - 7x}$ $(2 - x)^2 = (\sqrt{2 - 7x})^2$ $4 - 4x + x^2 = 2 - 7x$ $x^2 + 3x + 2 = 0$ $(x + 2)(x + 1) = 0$ $x = -2 \text{ or } x = -1$	4	
4.2	$x = 1 + \sqrt{7 - x}$ $(x - 1)^2 = (\sqrt{7 - x})^2$ $x^2 - 2x + 1 = 7 - x$ $x^2 - x - 6 = 0$ $(x - 3)(x + 2) = 0$ $x = 3 \text{ or } x = -2$ $x \neq 2$	5	
4.3	$x - 3\sqrt{x + 20} = 2$ $-(3\sqrt{x + 20})^2 = (-x + 2)^2$ $9(x + 20) = x^2 - 4x + 4$ $9x + 180 = x^2 - 4x + 4$ $x^2 - 13x - 176 = 0$ $(x - 16)(x + 11) = 0$ $x = 16 \text{ or } x = -11$ $x \neq -11$	4	
4.4	$x - 3\sqrt{x} - 4 = 0$ $(-3\sqrt{x})^2 = (-x + 4)^2$ $9x = x^2 - 8x + 16$ $x^2 - 17x + 16 = 0$ $(x - 16)(x - 1) = 0$ $x = 16 \text{ or } x = 1$ $x \neq 1$	4	
4.5	$\sqrt{x - 1} - 3\sqrt{x - 1} + 2 = 0$ $(-2\sqrt{x - 1})^2 = 2^2$ $4(x - 1) = 4$ $4x = 8$ $x = 2$	4	

4.6	$\sqrt{(7+3x)} + 2x = 0$ $\sqrt{(7+3x)} = -2x$ $7+3x = 4x^2$ $4x^2 - 3x - 7 = 0$ $(4x-7)(x+1) = 0$ $4x = \frac{7}{4} \text{ or } x+1 = 0$ $x = \frac{7}{4} \text{ or } x = -1$	5	
4.7	$(x+3) = \left(2\sqrt[3]{(5-x)}\right)^2$ $x^2 + 6x + 9 = 4(5-x)$ $x^2 + 10x - 11 = 0$ $(x-1)(x+11) = 0$ $x = 1 \text{ or } x = 11$	5	
4.8	$x - 2\sqrt{(x-1)} = 4$ $\left(-2\sqrt{(x-1)}\right)^2 = (4-x)^2$ $4(x-1) = 16 - 8x + x^2$ $x^2 - 12x + 20 = 0$ $(x-10)(x-2) = 0$ $x = 10 \text{ or } x = 2$	6	
4.9	$\sqrt{(x+5)} + 1 = x$ $\left(\sqrt{(x+5)}\right)^2 = (x-1)^2$ $x+5 = x^2 - 2x + 1$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $x = 4 \text{ or } x = -1$	3	
4.10	$x - \sqrt{x} = 6$ $\left(-\sqrt{x}\right)^2 = (-x+6)^2$ $x = x^2 - 12x + 36$ $x^2 - 13x + 36 = 0$ $(x-4)(x-9) = 0$ $x = 4 \text{ or } x = 9$ $x \neq 4$	4	
5.1	$(3-x)(x+1) < 0$ <i>cv</i> $x = 3 \text{ or } x = -1$ $\therefore 3 < x < -1$	4	
5.2	$x^2 - 16 \leq 0$ <i>cv</i>	4	

	$x^2 = 16$ $x = \pm 4$ $\therefore 4 \geq x \leq -4$		
5.3	$(x-1)(x-2) \leq 6$ $x^2 - 3x - 4 \leq 0$ cv $x = 4 \text{ or } x = 1$ $\therefore 4 \geq x \leq 1$	4	
5.4	$x^2 - 9x \geq 36$ $x^2 - 9x - 36 \geq 0$ cv $x = -3 \text{ or } x = 12$ $\therefore -3 \leq x \leq 12$	4	
5.5	$3x^2 + x - 2 \geq 0$ $cv (3x-2)(x+1) = 0$ $x = \frac{2}{3} \text{ or } x = -1$ $\therefore \frac{2}{3} \leq x \leq -1$	4	
5.6	$15x^2 + 4 > 21x$ $5x^2 - 21x + 4 > 0$ cv $(5x-1)(x-4) = 0$ $x = \frac{1}{5} \text{ or } x = 4$ $\therefore \frac{1}{5} < x < 4$	4	
5.7	$6x - 2x^2 \leq 0$ $-2x^2 + 6x \leq 0$ $-2x(x-3) \leq 0$ $0 \leq x \leq 3$	4	
5.8	$4x^2 - 1 < 0$ $(2x-1)(2x+1) < 0$ $-\frac{1}{2} < x < \frac{1}{2}$	4	
5.9	$-x^2 < -x + 12$ $x^2 - x - 12 > 0$ $(x-4)(x+3) > 0$ $-3 < x < 4$	4	
5.10	$-x^2 - 3x > 0$ $-x(x+3) > 0$ $-3 < x < 0$	4	
5.11	$(x-1)(x+4) \geq 6$ $x^2 + 4x - x - 4 - 6 \geq 0$ $x^2 + 3x - 10 \geq 0$	3	

	$(x+5)(x-2) \geq 0$ $-5 \leq x \leq 2$		
6.1	$\left(\frac{10^n + 2^{n+2}}{5^{2n} + 4 \cdot 5^n}\right)^{\frac{1}{n}}$ $= \left(\frac{5^n \cdot 2^n + 2^n \cdot 2^2}{5^{2n} + 4 \cdot 5^n}\right)^{\frac{1}{n}}$ $= \left(\frac{2^n(5^n + 4)}{5^n(5^n + 4)}\right)^{\frac{1}{n}}$ $= \left(\frac{2^n}{5^n}\right)^{\frac{1}{n}}$ $= \frac{2}{5}$	4	
6.2	$3^{x+2} - 3^{2-x} = 82$ $3^x \cdot 3^2 - 3^2 \cdot 3^{-x} = 82$ $9 \cdot 3^{2x} - 82 \cdot 3^x - 9 = 0$ $(9 \cdot 3^x - 1)(3^x - 9) = 0$ $x = -2 \text{ or } x = 2$	5	
6.3	$3^{x+1} - 3^{x-1} - 24 = 0$ $3^x \left(3 - \frac{1}{3}\right) = 24$ $3^x \left(\frac{8}{3}\right) = 24$ $3^x = 3^2$ $x = 2$	5	
6.4	$2^{2x} - 6(2^x) = 16$ $2^{2x} - 6(2^x) - 16 = 0$ $(2^x - 8)(2^x + 2) = 0$ $2^x = 8 \text{ or } 2^x = -2$ $x = 3$	4	

Solutions level 3

1	$\frac{\sqrt{10^{1009}}}{\sqrt{10^{1011}} - \sqrt{10^{1007}}}$	3
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	$\frac{\sqrt{10^{1007} 10^2}}{\sqrt{10^{1007} 10^4} - \sqrt{10^{1007}}}$ $\frac{\sqrt{\sqrt{10^{1007} 10^2}}}{\sqrt{(\sqrt{10^{1007}})(10^4 - 1)}}$ $-\frac{10}{99}$	
2	$\frac{\sqrt{3^{2011}} - \sqrt{3^{2009}}}{\sqrt{3^{2008}}}$ <p>Type equation here.</p> <p>Answer is 12</p>	4
3	$a = 2 \text{ or } b = 1006$	3
4	Type equation here.	
6	$2^x(x - 5) \leq 0$ $2^x \leq 0 \text{ or } x - 5 \leq 0$ $x \leq 0 \text{ or } x \leq 5$	3
7	$x^2 + 2xy - 8y^2 = 0$ $(x + 4y)(x - 2y) = 0$ $x + 4y = 0 \text{ or } x - 2y = 0$ $x = -4y \text{ or } x = 2y$ $\frac{x}{y} = -4 \text{ or } \frac{x}{y} = 2$ 7.2 $x + y = 6$ $\frac{x}{y} = 2$ $2y + y = 6$ $3y = 6$ $y = 2$	3 5
8	It is not possible for f(x) to be equal to g(x)	2
9	9.1 $\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right) = (4)^2$ $x^2 + 1 + 1 + \frac{1}{x^2} = 16$ $x^2 + \frac{1}{x^2} = 14$ 9.2 $\left(x + \frac{1}{x}\right)\left(x^2 - 1 + \frac{1}{x^2}\right)$ $= (4)(14 - 1)$ $= 4(13)$ $= 52$	2 3
10.1	$(3x - y)^2 + (x - 5)^2 = 0$ $3x - y = 0 \text{ or } x - 5 = 0$ $x = 5 \text{ then } y = 15$	6

Question 2 level 4

1	$6^x + 6^x + 6^x + 6^x + 6^x + 6^x = 6^{6x}$ $6 \cdot 6^x = 6^{6x}$ $6 = 6^{6x-x}$ $6 = 6^{5x}$ $5x = 1$ $x = \frac{1}{5}$	3
2	$x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$ $\left(x^{\frac{1}{4}} + 6\right)\left(x^{\frac{1}{4}} - 3\right) = 0$ $x^{\frac{1}{4}} \neq -6 \text{ or } x = 81$	4
3	$3^{x+1} + m \cdot 3^x = 2m + 6$ $3^x(3 + m) = 2(m + 2)$ $3^x = 2$ $x \log(3) = \log(2)$ $x = \frac{\log(2)}{\log(3)}$	4
4	$9 \cdot 3^{2x} + 1 = 6 \cdot 3^x$ $\text{let } k = 3^x$ $9k^2 + 1 = 6 \cdot k$ $9k^2 - 6k + 1 = 0$ $(3k - 1)(3k - 1) = 0$ $k = \frac{1}{3}$ $3^x = \frac{1}{3}$ $x = -1$	(4)
5.1	$2y = 3 + x \text{ --- (1)}$ $2xy + 7 = x^2 + 4y^2 \text{ --- (2)}$ $x = 2y - 3 \text{ --- (3)}$ sub eq3 into eq2	(5)

	$2y(2y - 3) = (2y - 3)^2 + 4y^2$ $4y^2 - 6y + 2 = 0$ $(2y - 1)(y - 1) = 0$ $y = \frac{1}{2} \text{ or } y = 1$ $\therefore x = -2 \text{ or } x = -1$	
5.2	$(3x - y)^2 + (x + 5)^2 = 0$ $(3x - y)^2 = 0 \text{ and } (x + 5)^2 = 0$ $3x - y = 0 \text{ and } x + 5 = 0$ $3x = y$ $x = -5$ $y = 3(-5)$ $y = -15$	3
5.3	$2^{x+1} + 2^x = 3^{y+2} - 3^y$ $2^x \cdot 2 + 2^x = 3^y \cdot 3^2 - 3^y$ $2^x(2 + 1) = 3^y(9 - 1)$ $2^x \cdot 3 = 3^y \cdot 8$ $\frac{2^x}{3^y} = \frac{8}{3}$ $2^x = 2^3$ $x = 3 \text{ or } 3^y = 3$ $y = 1$	3
5.4	$2^x - 2^{y+2} = 0 \dots\dots\dots 1$ $x^2 + 2xy + y^2 = 0 \dots\dots\dots 2$ $2^x = 2^{y+2}$ $x = y + 2$ $(x + y)^2 = 0$ $x + y = 0$ $x = -y$ $x = y + 2$ $-y = y + 2$ $-2y = 2$ $y = -1$ $x = y + 2$ $x = -1 + 2$ $x = 1$	3
6.1	$f(x) = 3(x - 1)^2 + 5$ $g(x) = 3$ $f(x) = g(x)$ $3(x - 1)^2 + 5 = 3$ $3(x - 1)^2 = -2$ $(x - 1)^2 = -\frac{2}{3}$ $x - 1 = \pm \sqrt{-\frac{2}{3}}$ non-real $\therefore \text{it is not possible for } f(x) = g(x)$	3
6.2	$f(x) = g(x) + k$ $3(x - 1)^2 + 5 = 3 + k$	4

	$3(x^2 - 2x + 1) + 5 = 3 + k$ $3x^2 - 6x + 3 + 5 = 3 + k$ $3x^2 - 6x + 8 = 3 + k$ $3x^2 - 6x + 5 - k = 0$ $\Delta = 0$ $b^2 - 4ac = 0$ $(-6)^2 - 4(3)(5 - k) = 0$ $36 - 60 + 12k = 0$ $-24 + 12k = 0$ $-24 + 12k = 0$ $12k = 24$ $k = 2$	
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SOLUTIONS FOR NUMBER PATTERN, SEQUENCE AND SERIES

QUESTION 1

1.1 $11\checkmark$

1.2 $T_n = a + (n - 1)d$
 $= 20 + (n - 1)(-3) \checkmark$
 $= 23 - 2n \checkmark$

1.3 $-103 = 23 - 3n$

$$\begin{aligned} -126 &= -3n \checkmark \\ n &= 42 \checkmark \end{aligned}$$

QUESTION 2

2.1 $35\checkmark$

2.2 $T_n = an^2 + bn + c$

$$2a = 4 \qquad 3a + b = 2$$

$$a + b + c = 3$$

$$a = 2 \checkmark \qquad 3(2) + b = 2$$

$$2 - 4 + c = 3$$

$$b = -4 \checkmark$$

$$c = 5 \checkmark$$

$$T_n = 2n^2 - 4n + 5$$

$$\begin{aligned} T_{48} &= 2(48)^2 - 4(48) + 5 \checkmark \\ &= 4421 \checkmark \end{aligned}$$

QUESTION 3

3.1 $b - 3 = 27 - 19\checkmark$

$$b = 11 \checkmark$$

3.2 $T_n = a + (n - 1)d$

$$= 3 + (n - 1)(8) \checkmark$$

$$= 8n - 5 \checkmark \checkmark$$

3.3 $T_{13} = 8(13) - 5 \checkmark$

$$= 99 \checkmark$$

3.4 $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{30} = \frac{30}{2}[2(3) + (30 - 1)8] \checkmark \checkmark$$

$$= 3570 \checkmark$$

QUESTION 4

$$4.1 \sum_{n=1}^{20} 3^{n-2} = 3^{-1} + 3^0 + \dots + 3^{18}$$

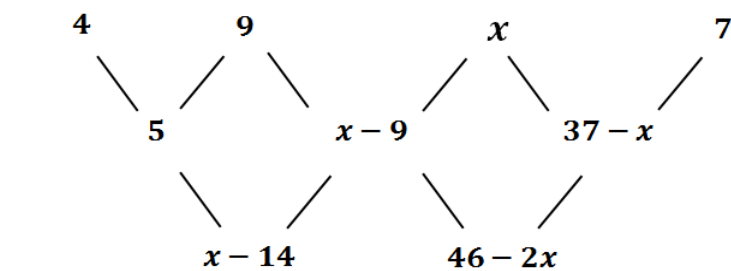
$$\therefore a = 3^{-1}; r = 3; n = 20 \checkmark$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\begin{aligned} S_{20} &= \frac{\frac{1}{3}(3^{20}-1)}{3-1} \checkmark \\ &= \frac{1}{3} \left(\frac{3^{20}-1}{2} \right) \checkmark \\ &= 581\,130\,733.3 \checkmark \end{aligned}$$

QUESTION 5

5.1 Quadratic $\therefore 2^{\text{nd}}$ difference constant.



$$\therefore x - 14 = 46 - 2x \checkmark$$

$$3x = 60 \checkmark$$

$$x = 20 \checkmark$$

(3)

5.2 $x = 20; x - 14 = 6$

$$\therefore 2a = 6 \therefore a = 3 \checkmark$$

$$\therefore T_n = an^2 + bn + c$$

$$(a = 3) \therefore T_n = 3n^2 + bn + c$$

$$n = 1: \therefore 4 = 3(1)^2 + (1) + c$$

$$= 3 + b + c$$

$$\therefore 1 = b + c \quad \text{①}$$

$$n = 2: 9 = 3(2)^2 + (2) + c \checkmark$$

$$= 12 + 2b + c$$

$$\therefore -3 = 2b + c \quad \text{②}$$

$$\text{①}-\text{②}: 4 = -b$$

$$\therefore b = -4 \checkmark$$

$$\text{in } \textcircled{1}: \therefore 1 = (-4) + c$$

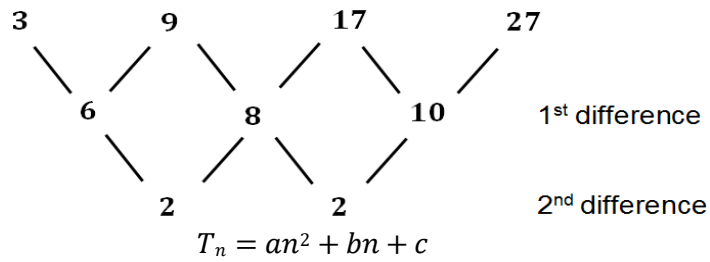
$$\therefore 5 = c \checkmark$$

$$\therefore T_n = 3n^2 - 4n + 5 \quad (4)$$

QUESTION 6

$$6.1 \ 3; 9; 17; 27; 39 \quad \checkmark \quad (1)$$

6.2



$$\therefore 2a = 2 \checkmark$$

$$\therefore a = 1 \checkmark$$

$$\therefore T_n = n^2 + bn + c$$

$$n = 1: T_1 = 1^2 + (1) + c = 3$$

$$\therefore b + c = 2 \quad \textcircled{1}$$

$$n = 2: T_2 = 2^2 + (2) + c = 9$$

$$\therefore 2b + c = 5 \quad \textcircled{2}$$

$$\textcircled{1} - \textcircled{2}: -b = -3$$

$$\therefore b = 3 \checkmark \text{ in } \textcircled{1}: 3 + c = 2$$

$$\therefore c = -1 \checkmark$$

$$\therefore T_n = n^2 + 3n - 1 \quad (4)$$

QUESTION 7

$$7.1 \ 4; x; 32$$

$$7.1.1 \ d = x - 4 = 32 - x \checkmark$$

$$\begin{aligned} \therefore 2x &= 36 \\ x &= 18 \quad \checkmark \end{aligned}$$

(2)

$$7.1.2 \ r = \frac{x}{4} = \frac{32}{x} \checkmark$$

$$\therefore x^2 = 128 \checkmark$$

$$\therefore x = \pm \sqrt{128}$$

$$= \pm \sqrt{2} \checkmark$$

$$\begin{aligned}
 7.2 \quad P &= \sum_{k=1}^{13} 3^{k-5} \\
 &= 3^{-4} + 3^{-3} + \dots + 3^3 \checkmark \\
 \therefore a &= 3^{-4}; r = 3; n = 13 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 S_n &= \frac{a(r^n - 1)}{r - 1} \\
 &= \frac{3^{-4}(3^{13} - 1)}{3 - 1} \checkmark \\
 &= \frac{3^{13} - 1}{3^4 \times 2} \\
 &= 9841,49 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 7.3 \quad S_n &= a + (a + d) + \dots + [a + (n - 1)d] \\
 S_n &= [a + (n - 1)d] + [a + (n - 2)d] + \dots + a \checkmark \\
 2S_n &= [2a + (n - 1)d] + [2a + (n - 2)d] + \dots + [2a + (n - 1)d] \checkmark \\
 \therefore 2S_n &= n \times [2a + (n - 1)d] \checkmark \\
 S_n &= \frac{n}{2}[2a + (n - 1)d] \checkmark
 \end{aligned}$$

QUESTION 8

$$8.1 \quad d = T_2 - T_1 = -3 - (-7) = 4$$

$$\begin{aligned}
 T_n &= a + (n - 1)d \\
 \therefore 173 &= -7 + (n - 1)(4) \quad \checkmark
 \end{aligned}$$

$$173 = -7 + 4n - 4$$

$$184 = 4n \checkmark$$

$$n = 46 \text{ terms } \checkmark$$

(3)

$$\begin{aligned}
 8.2 \quad S_n &= \frac{n}{2}(a + l) \\
 S_{46} &= \frac{46}{2}(-7 + 173) \checkmark \\
 &= 23(166) \checkmark \\
 &= 3818 \checkmark
 \end{aligned}$$

$$\begin{aligned}
 8.3 \quad T_n &= a + (n - 1)d \\
 &= -7 + (n - 1)(4) \quad \checkmark \\
 &= -7 + 4n - 4 \\
 &= 4n - 11 \checkmark \\
 \therefore -7 - 3 + 1 + \dots + 173 &= \sum_{n=1}^{46} (4n - 11) \checkmark
 \end{aligned}$$

QUESTION 9

9.1

$$9.1.1 \quad r = \frac{-2}{4} = \frac{-1}{2} \checkmark$$

$$\therefore T_4 = 1 \times -\frac{1}{2} = -\frac{1}{2} \checkmark$$

$$9.1.2 \quad T_{n=ar^{n-1}}$$

$$\frac{1}{64} = 4\left(-\frac{1}{2}\right)^{n-1} \checkmark$$

$$\left(-\frac{1}{2}\right)^8 = \left(-\frac{1}{2}\right)^{n-1} \checkmark$$

$$\therefore 8 = n - 1 \checkmark$$

$$\therefore n = 9 \checkmark$$

$$9.1.3 \quad S_{\infty} = \frac{a}{1-r}$$

$$= \frac{4}{1 - \left(-\frac{1}{2}\right)} \checkmark$$

$$= \frac{8}{3} \checkmark$$

9.2 For Geometric sequence.

$$\frac{x+1}{1} = \frac{x-3}{x+1}$$

$$\therefore x^2 + 2x + 1 = x - 3$$

$$\therefore x^2 + x + 4 = 0 \checkmark$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 - 16}}{2}$$

$$= \frac{-1 \pm \sqrt{-15}}{2} \checkmark$$

\therefore No real solution. \checkmark

\therefore Not a geometric sequence. \checkmark

QUESTION 10

10.1

$$10.1.1 S_n = n^2 + 3n$$

$$S_1 = 1^2 + 3(1) \checkmark$$

$$= 1 + 3$$

$$= 4$$

$$\therefore T_1 = 4 \checkmark$$

$$S_2 = 2^2 + 3(2) = 4 + 6 = 10$$

$$\therefore T_2 = S_2 - S_1 = 10 - 4 = 6 \checkmark$$

10.1.2 $4 + 6 + 9 + \dots$

$$a = 4; r = \frac{3}{2} \checkmark$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_6 = \frac{4\left(\left(\frac{3}{2}\right)^6 - 1\right)}{\frac{3}{2} - 1} \checkmark$$

$$= 83.13 \checkmark$$

$$10.2 \quad T_1 = 6\left(\frac{1}{2}\right)^1 = 3 \checkmark$$

$$T_2 = 6\left(\frac{1}{2}\right)^2 = \frac{3}{2}$$

$$T_3 = 6\left(\frac{1}{2}\right)^3 = \frac{3}{4}$$

$$\therefore a = 3; r = \frac{1}{2} \checkmark$$

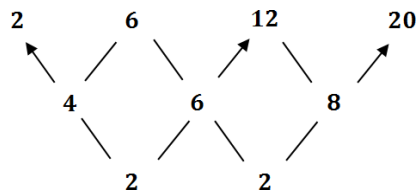
$$S_\infty = \frac{a}{1 - r}$$

$$S_\infty = \frac{3}{1 - \frac{1}{2}} \checkmark$$

$$= 6 \checkmark$$

QUESTION 11

11.1



$$2a = 2 \checkmark$$

$$\therefore a = 1 \checkmark$$

$$T_n = 1n^2 + bn + c$$

$$T_2 = 2^2 + (2) + c$$

$$\therefore 6 = 4 + 2b + c$$

$$2 = 2b + c$$

$$T_1 = 1^2 + b(1) + c$$

$$2 = 1 + b + c$$

$$1 = b + c$$

$$\textcircled{1}-\textcircled{2}: 1 = b \checkmark$$

$$b = 1 \text{ in } \textcircled{2}: 1 = 1 + c$$

$$\therefore c = 0 \checkmark$$

$$\therefore a = 1; b = 1; c = 0$$

$$11.2 \quad T_n = n^2 + n$$

$$T_{12} = 12^2 + 12$$

$$= 144 + 12 \checkmark$$

$$= 156 \checkmark$$

(2)

QUESTION 12

$$12.1 \quad r = \frac{1}{2} \text{ and } s_{\infty} = 6$$

$$S_{\infty} = \frac{a}{1-r}$$

$$6 = \frac{a}{1-\frac{1}{2}} \checkmark$$

$$a = 3 \checkmark$$

$$12.2 \quad T_n = ar^{n-1}$$

$$T_8 = 3 \left(\frac{1}{2} \right)^{8-1} \checkmark$$

$$= \frac{3}{128} \checkmark$$

$$12.3. \quad \sum_{k=1}^n 3(2)^{1-k} = 5,8125$$

$$3 + \frac{3}{2} + \frac{3}{4} + \dots = 5.8125 \checkmark$$

$$S_n = \frac{a(1-r^n)}{1-r} = 5,8125$$

$$\frac{3[1-(\frac{1}{2})^n]}{1-\frac{1}{2}} = 5.8125 \checkmark$$

$$6[1 - (\frac{1}{2})^n] = 5.8125$$

$$(\frac{1}{2})^n = \frac{1}{32} = 0.03125$$

$$2^{-n} = 2^{-5} \text{ or } n \log \frac{1}{2} = \log \frac{1}{32} \checkmark$$

$$\therefore n = 5 \checkmark$$

QUESTION 13

$$13.1 \quad 1 - \frac{1}{4}; b; -1$$

$$\frac{b}{-\frac{1}{4}} = -\frac{1}{b} \checkmark$$

$$b^2 = \frac{1}{4} \checkmark$$

$$b = \pm \frac{1}{2} \checkmark$$

$$13.2 \quad -\frac{1}{4}; \frac{1}{2}; -1$$

$$r = -2$$

$$T_{19} = ar^{18}$$

$$T_{19} = \left(-\frac{1}{4}\right)(-2)^{18} \checkmark$$

$$= \left(\frac{2^{18}}{2^2}\right)$$

$$= -2^{16} \checkmark$$

$$= -65536 \checkmark$$

$$13.3 \quad \text{The series is } -\frac{1}{4}; \frac{1}{2}; -1; 2; -4; 8; \dots$$

$$\text{The new positive term series: } \frac{1}{2}; 2; 8; 32; 128; \dots \checkmark$$

$$a = \frac{1}{2}; r = 4 \checkmark$$

$$\sum_{n=1}^{20} \left(\frac{1}{2}\right)(4)^{n-1} \checkmark \checkmark$$

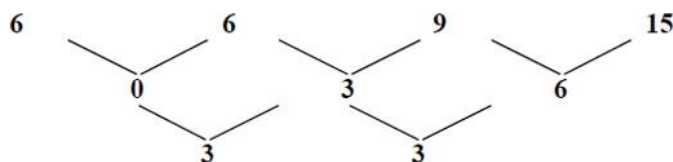
$$13.4 \text{ No, the series is not convergent } \checkmark$$

$$r = 4 \text{ and for convergence } -1 < r < 1 \checkmark$$

QUESTION 14

$$14.1.1 \quad 24 \quad \checkmark$$

$$14.1.2$$



$$2a = 3$$

$$3a + b = 0$$

$$a + b + c = 6$$

$$a = \frac{3}{2} \checkmark$$

$$b = -\frac{9}{2} \checkmark$$

$$c = 9 \checkmark$$

$$T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$$

$$14.1.3 \quad \frac{3}{2}n^2 - \frac{9}{2}n + 9 = 3249 \checkmark$$

$$3n^2 - 9n + 18 = 6498$$

$$3n^2 - 9n - 6480 = 0$$

$$n^2 - 3n - 2160 = 0 \checkmark$$

$$(n + 45)(n - 48) = 0 \checkmark$$

$$n \neq -45 \text{ or } n = 48 \checkmark$$

QUESTION 15

$$15.1 \quad \frac{2}{5 \times 6}; \frac{2}{6 \times 7} \checkmark \checkmark$$

$$15.2 \quad T_n = \frac{2}{n \times (n+1)} = \frac{2}{n^2+n} \checkmark \checkmark$$

$$15.3 \quad T_n = \frac{2}{n^2+n}$$

$$\therefore \frac{1}{66} = \frac{2}{n^2+n} \checkmark$$

$$n^2 + 17 = 132 \checkmark$$

$$n^2 + n - 132 = 0 \checkmark$$

$$(n + 12)(n - 11) = 0$$

$$\therefore n \neq -12 \text{ or } n = 11$$

SOLUTIONS FOR LEVEL 3 AND 4 QUESTIONS

Question 1

1.1

$$T_1 = 8 \text{ and } T_2 = \frac{8}{\sqrt{2}}$$

$$r = \frac{T_2}{T_1} = \frac{\frac{8}{\sqrt{2}}}{8} = \frac{8}{\sqrt{2}} \times \frac{1}{8} = \frac{1}{\sqrt{2}}$$

$$S_\infty = \frac{a}{1-r} = \frac{8}{1-\frac{1}{\sqrt{2}}} = \frac{8}{\frac{\sqrt{2}-1}{\sqrt{2}}}$$

$$= \frac{8\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2})^2 - 1^2}$$

$$\frac{8(2)+8\sqrt{2}}{2-1}$$

$$16 + 8\sqrt{2} \quad \checkmark$$

1.2

1.2.1

$$x = 5 + 15 + 45 + \dots + 20\text{term}$$

$$\therefore a = 5$$

$$\therefore r = \frac{15}{5} = 3 \quad \checkmark$$

$$\therefore n = 20$$

$$\therefore x = \sum_{k=1}^n ar^{k-1} = \sum_{k=1}^{20} 5(3)^{k-1} \quad \checkmark$$

1.2.2

$$S_N = \frac{a(r^n - 1)}{r - 1}$$

$$x = s_{20} = \frac{5(3^{20} - 1)}{3 - 1} \quad \checkmark \checkmark$$

$$x = 8716961000 \quad \checkmark$$

Question 2

2.1

$$s_n = \frac{n}{2} (5n + 9)$$

$$\therefore s_{23} = \frac{23}{2} (5(23) + 9) = 1426 \quad \checkmark \checkmark$$

$$\begin{aligned} T_{23} &= S_{23} - S_{22} \\ &= 1426 - \frac{22}{2} (5 \times 22 + 9) \quad \checkmark \checkmark \\ &= 117 \quad \checkmark \end{aligned}$$

2.2 Arithmetic sequence: $a, a+d, a+2d$

Geometric sequence: a, ar, ar^2

$$\therefore a + d = ar \dots (T_2)$$

$$a = 12 : 12 + d = 12r \quad d =$$

$$(12r - 12) \dots \dots \dots 1 \quad \checkmark$$

$$\text{and } (s_3)_{GS} - 3 = (S_3)_{AS}$$

$$\therefore 12 + 12r + 12r^2 - 3 = 12 + 12 + d + 12 + 12d$$

$$\therefore 12r^2 + 12r + 9 = 36 + 3d \quad (2)$$

$$\text{sub(1) into (2)}$$

$$12r^2 + 12r + 9 = 36 + 3(12r - 12) \quad \checkmark$$

$$\therefore 12r^2 - 24r + 9 = 0 \quad \checkmark$$

$$4r^2 - 8r + 3 = 0$$

$$\therefore (2r - 1)(2r - 3) = 0 \quad \checkmark$$

$$\therefore r = \frac{1}{2} \text{ or } \frac{3}{2} \quad \checkmark \checkmark$$

Question 3

$$3.1 \quad T_{52}: \text{Geometric } T_{26}$$

$$T_{51}: \text{Arithmetic } T_{26}$$

$$\therefore T_{52} : T_n = ar^{n-1}$$

$$\therefore T_{26} = 3.2^{26-1} = 3.2^{25} \checkmark$$

$$= 100663296 \checkmark$$

$$T_{26} = 3 + (26-1)6$$

$$T_{26} = 3 + 25 \times 6 \checkmark$$

$$T_{26} = 153 \checkmark$$

$$T_{52} - T_{51} = 100663296 - 153 = 100663143 \checkmark$$

$$3.2 \text{ Arithmetic: } T_{K=a+(k-1)d}$$

$$= 3 + (k-1)6$$

$$= 3 + 6k - 6$$

$$= 3(k-2) \checkmark$$

$$\text{Geometric: } = ar^{k-1}$$

$$= 3.2^{k-1} \checkmark$$

Both divisible by 3

Question 4

$$4.4 \quad d(n) = n^2 - 10n + 26$$

$$d'(n) = 2n - 10$$

$$\text{If } n < 5 \text{ then } 2n - 10 < 0$$

$$\therefore d'(n) < 0$$

The distance is decreasing, moving towards P

QUESTION 5

$$5.2 \quad -1; 2\sin 3x; 5; \dots$$

$$2\sin 3x + 1 = 5 - 2\sin 3x \quad \checkmark$$

$$4\sin 3x = 4 \quad \checkmark$$

$$\sin 3x = 1 \quad \checkmark$$

$$3x = 90^\circ \quad x = 13 \quad \checkmark$$

Question 6

$$6.1 \quad 9 \quad \checkmark$$

$$6.2 \quad 25 \quad \checkmark$$

$$6.3 \quad T_n = 2n - 1 \quad \checkmark \quad \checkmark$$

$$6.4 \quad T_n = (n - 1)^2 \quad \checkmark \quad \checkmark$$

$$6.5 \quad 64 = (n - 1)^2$$

$$n^2 - 2n - 63 = 0 \quad \checkmark$$

$$(n - 9)(n + 7) = 0 \quad \checkmark$$

$$n = 9 \text{ or } n = -7$$

$$T_9 = 64 \quad \checkmark$$

$$6.6 \quad \text{Area} = 100(s_n)$$

$$\text{Area} = 100\left(\frac{100}{2}[2(1) + (100 - 1)(2)]\right) \quad \checkmark \quad \checkmark$$

$$= 1000000 \text{ m}^2 \quad \checkmark$$

QUESTION 7

$$7.1 \quad -14, 0, 10 \quad \checkmark \checkmark \checkmark$$

$$7.2 \quad T_5 = 18 \quad \checkmark$$

$$7.3 \quad d = -4 \quad \checkmark \checkmark$$

$$7.4 \quad a = -2 \quad \checkmark$$

$$3(-2) + b = 14$$

$$b = 20 \quad \checkmark$$

$$-2 + 20 + c = -14$$

$$c = -32 \quad \checkmark$$

$$-110 = -2n + 20n - 32$$

$$n^2 - 10n - 39 = 0 \quad \checkmark$$

$$(n+3)(n-13) = 0 \quad \checkmark$$

$$n = -3 \text{ or } n = 13 \quad \checkmark$$

$$n = 13 \quad \checkmark$$

Question 8

$$8.4 \quad d_1: 14; 18; 22$$

$$110 = 14 + (n-1)(4) \quad \checkmark$$

$$n = 25 \quad \checkmark$$

$$T_{25} = 1453 \quad \checkmark$$

$$T_{26} = 1563 \quad \checkmark$$

$$8.5 \quad T_n = 2n^2 + 8n + 3$$

$$T_n = \text{even number} + \text{even number} + \text{odd number} \quad \checkmark$$

$$T_n = \text{even number} + \text{odd number}$$

$$T_n = \text{odd number} \quad \checkmark$$

Question 9

$$9.2 \text{ sum of area} = [0] + [1 - (\frac{1}{2})^2] + [1 - (\frac{1}{4})^2] + [1 - (\frac{1}{8})^2] + [1 - (\frac{1}{32})^2] + [1 - (\frac{1}{64})^2] \quad \checkmark\checkmark\checkmark$$

$$\text{Sum of area} = 0 + \frac{3}{4} + \frac{15}{16} + \frac{63}{64} + \frac{255}{256} + \frac{1023}{1024} + \frac{4095}{4096} \quad \checkmark$$

$$\text{Sum of area} = 5.67 \text{ units}^2 \quad \checkmark$$

Question 10

10.1

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots, 19 \text{ terms}$$

$$\text{GS: } a = \frac{1}{2}; \quad r = \frac{1}{2}; \quad n = 19$$

$$\frac{1}{2} \quad \frac{1}{2}$$

$$= s_{19} = \frac{\frac{1}{2}(1 - \frac{1^{19}}{2})}{1 - \frac{1}{2}} = \frac{524287}{524288} < 1$$

Yes, all will fit into 1st tank

Question 11

11.1

$$s_n = \frac{n}{2}(2a + (n-1)d) \quad \checkmark$$

$$s_8 = \frac{8}{2}(2(88) + (8-1)(4)) = 816 \quad \checkmark \quad \checkmark$$

$$11.2 \quad \text{mean} = \frac{20}{2}(2(136) + (20-1)(4)) \div 2 \quad \checkmark$$
$$\text{mean} = 1740 \quad \checkmark$$

Question 12

$$12.1 \quad \frac{1}{16}; 13 \quad \checkmark \quad \checkmark$$

1.2.2

$$s_{50} = \frac{a(1-r^n)}{1-r} + \frac{n}{2}[2a + (n-1)d]$$

$$s_{50} = \frac{\left(\frac{1}{2}\right)\left(1 - \left(\frac{1}{2}\right)^{25}\right)}{1 - \frac{1}{2}} + \frac{25}{2}[2(4) + (25-1)(6)] \quad \checkmark \quad \checkmark$$

$$s_{50} = 1901 \quad \checkmark \quad \checkmark$$

Question 13

13.1

$$0,89x; 0,89^2x; 0,89^3x; \dots \quad \checkmark$$

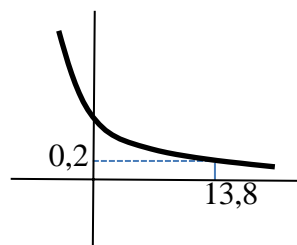
$$T_n = ar^{n-1}$$

$$= 0,89x (0,89)^{n-1} \quad \checkmark$$

$$= x (0,89)^n$$

$$0,2x = x (0,89)^n$$

$$0,2 = (0,89)^n$$



$$n = \log_{0,89} 0,2 \quad \checkmark$$

$$n = 13,8 \quad \checkmark$$

∴ 13 copies can be made
✓

Question 14

14.1 8; 4; 2;

$$a = 8 \quad r = \frac{1}{2} \quad \checkmark \quad \checkmark$$

$$T_{15} = ar^{14}$$

$$T_n = 8\left(\frac{1}{2}\right)^{15-1} \quad \checkmark$$

$$= \frac{1}{2048} \quad \checkmark$$

14.2

$$OC = 16 + 8 + 4 + \dots \quad \checkmark$$

$$OC = S_{\infty} = \frac{16}{1 - \frac{1}{2}} \quad \checkmark \quad \checkmark = 32$$

14.3 ABC=90 [tan ⊥ radius] ✓

14.4

$$BC = \sqrt{24^2 - 8^2} \quad \checkmark$$

$$= 16\sqrt{2} \quad \checkmark$$

$$\tan \theta = \frac{8}{16\sqrt{2}} = \frac{\sqrt{2}}{4} \quad \checkmark$$

!!!!!!!!!!

THE END

!!!!!!!!!!

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

QUESTION 1

1.1	$x = -\frac{b}{2a} \quad 2x - 4 = 0$ $= -\frac{(-4)}{2(1)} \quad \text{OR / OF} \quad 2x = 4$ $= 2 \quad x = 2$ $y = (2)^2 - 4(2) - 11$ $= -15$ $D(2; -15)$	<p>✓ subst. into correct formula verv. in korrekte formule (method mark / metodepunt)</p> <p>✓ x-value / x-waarde</p> <p>✓ y-value / y-waarde</p> <p>(3)</p>
1.2	$g(x) = f'(x) = 2x - 4$ <p>coordinates of C / koördinate van C :</p> $C(2; 0)$ <p style="text-align: center;">OR/OF</p> <p>Making connection between x-coordinate of T/P of the function and the x-intercept of the derivative of the function. Concluding that $C(2; 0)$.</p> <p><i>Maak konneksie tussen x-koördinaat van draaipunt van die funksie en die x-afsnit van die afgeleide van die funksie. Gevolglik is $C(2; 0)$.</i></p> $CN = \sqrt{(7-2)^2 + (10-0)^2}$ $= \sqrt{125}$ $= 5\sqrt{5}$	<p>✓✓ coordinates of C koördinate van C</p> <p>✓ substitution / vervanging</p> <p>✓ answer / antwoord (4)</p>
1.3	$-1 < x < 7$	<p>✓✓ answer / antwoord</p> <p>(2)</p>
1.4	$g(x) - f(x)$ $= 2x - 4 - (x^2 - 4x - 11)$ $= -x^2 + 6x + 7$ <p>For maximum / Vir maksimum: $-2x + 6 = 0$</p> $\therefore x = 3$	<p>✓ difference / verskil</p> <p>✓ derivative / afgeleide ✓ equating derivative to 0 stel afgeleide = 0</p> <p>✓ answer / antwoord</p> <p>(4)</p>
		[13]

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

QUESTION 2

2.1	$f(x) = -x^2 - 6x + 7$ $f'(x) = -2x - 6$ $-2x - 6 = 0$ OR/OF $x = -\frac{(-6)}{2(-1)}$ $x = -3$ $E(-3 ; 16)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;">ANSWER ONLY: FULL MARKS</div>	✓ method ✓ x-value ✓ y-value (3)
2.2	$k = f(-5)$ $k = -(-5)^2 - 6(-5) + 7$ $\therefore k = 12$	✓ answer (A) (1)
2.3	$C(0 ; 7)$ $D(-5 ; 12)$ $m_{CD} = \frac{12 - 7}{-5 - 0}$ $m_{CD} = -1$ Equation of CD: $y = -x + 7$	✓ coordinates of C ✓ substitution ✓ m ✓ answer (4)
2.4	$-2x - 6 = -1$ $-2x = 5$ $x = -\frac{5}{2}$ $y = f\left(-\frac{5}{2}\right) = -\left(-\frac{5}{2}\right)^2 - 6\left(-\frac{5}{2}\right) + 7 = \frac{63}{4} = 15,75$ $\therefore P\left(-\frac{5}{2}; \frac{63}{4}\right)$	✓ $f'(x) = -2x - 6$ ✓ equating to -1 ✓ x-value ✓ y-value (A) (4)
2.5	Point by symmetry: $(-1 ; 12)$ $-5 < x < -1$ OR/OF $-x^2 - 6x + 7 > 12$ $-x^2 - 6x - 5 > 0$ $x^2 + 6x + 5 < 0$ $(x+1)(x+5) < 0$ $-5 < x < -1$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;">ANSWER ONLY: FULL MARKS</div>	✓ -1 ✓ answer (2) (2)
		[14]

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

QUESTION 3		
3.1	$f(x) = -ax^2 + bx + 6$ $f'(x) = -2ax + b$ $f'(-1) = -2a(-1) + b$ $f'(-1) = 2a + b$ $3 - 2a = b \quad \dots(1)$ $f(x) = -ax^2 + bx + 6$ $\frac{7}{2} = -a(-1)^2 + b(-1) + 6$ $\frac{7}{2} = -a - b + 6$ $b = -a + \frac{5}{2} \quad \dots(2)$ $(1) = (2)$ $3 - 2a = -a + \frac{5}{2}$ $a = \frac{1}{2}$ $\therefore b = 3 - 2\left(\frac{1}{2}\right)$ $b = 2$	<p>✓ derivative/afgeleide</p> <p>✓ equation/vergelýking (1)</p> <p>✓ equation/vergelýking (2)</p> <p>✓ equating/gelykstel</p> <p>✓ substitution of a-value in any correct equation for b/vervanging van a-waarde in enige korrekte vergelyking om b te bepaal</p>
	OR/OF	OR/OF

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

	$f(x) = -ax^2 + bx + 6$ $f'(x) = -2ax + b$ $f'(-1) = -2a(-1) + b$ $\therefore 3 = 2a + b \dots\dots\dots(1)$ $f(x) = -ax^2 + bx + 6$ $(-1; \frac{7}{2}): \frac{7}{2} = -a(-1)^2 + b(-1) + 6$ $\frac{7}{2} - 6 = -a - b$ $-\frac{5}{2} = -a - b \dots\dots\dots(2)$ $(1) + (2): a = \frac{1}{2} \dots\dots\dots(3)$ $(3) \text{ in } (1): 3 = 2(\frac{1}{2}) + b$ $3 = 1 + b$ $b = 2$	<p>✓ derivative/afgeleide</p> <p>✓ equation/vergeliking (1)</p> <p>✓ equation/vergeliking (2)</p> <p>✓ method to determine <i>a</i>/ metode om <i>a</i> te bepaal</p> <p>✓ substitution of <i>a</i>-value in correct equation for <i>b</i>/ vervanging van <i>a</i>-waarde in korrekte vergelyking om <i>b</i> te bepaal</p> <p>(5)</p>
3.2	$f(x) = -\frac{1}{2}x^2 + 2x + 6$ x – intercepts/afsnit $\therefore f(x) = 0$ $0 = -\frac{1}{2}x^2 + 2x + 6$ $0 = x^2 - 4x - 12$ $0 = (x - 6)(x + 2)$ $\therefore x = 6 \text{ or/of } x = -2$ $(6; 0) \text{ or/of } (-2; 0)$ NOTE: Answer does not have to be in coordinate form./ LET WEL: Antwoord hoef nie in koördinaatvorm te wees nie	<p>✓ factors/faktore</p> <p>✓ answers/antwoorde</p> <p>(2)</p>

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

<p>3.3 $f(x) = -\frac{1}{2}x^2 + 2x + 6$ $f'(x) = -x + 2$ $0 = -x + 2$ $x = 2$ $f(2) = -\frac{1}{2}(2)^2 + 2(2) + 6$ $f(2) = 8$ \therefore TP (2;8)</p> <p>OR/OF</p> <p>$f(x) = -\frac{1}{2}x^2 + 2x + 6$ $x = \frac{-(2)}{2(-\frac{1}{2})}$ $x = 2$ $f(2) = -\frac{1}{2}(2)^2 + 2(2) + 6$ $f(2) = 8$ TP (2;8)</p> <p>OR/OF</p> <p>$x = \frac{-2+6}{2}$ $x = 2$ $f(2) = -\frac{1}{2}(2)^2 + 2(2) + 6$ $f(2) = 8$ TP (2;8)</p> <p>NOTE: Answer does not have to be in coordinate form./ LET WEL: Antwoord hoef nie in koördinaatvorm te wees nie</p>	<p>✓ derivative = 0/ afgeleide = 0 ✓ x- value/x-waarde</p> <p>✓ y- value/y-waarde</p> <p>OR/OF</p> <p>✓ correct subst. into formula/korrekte vervanging in formule ✓ x-value/x-waarde</p> <p>✓ y- value/y-waarde</p> <p>OR/OF</p> <p>✓ midpoint between x-intercepts/middelpunt tussen x-afsnitte ✓ x- value/x-waarde</p> <p>✓ y-value/y-waarde</p> <p>(3)</p>
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FUNCTIONS LEVEL 1 & 2 SOLUTIONS

QUESTION 4/VRAAG 4

4.1	$\frac{m-5+m+3}{2} = 2$ $2m-2=4$ $2m=6$ $\therefore m=3$	$\checkmark \frac{m-5+m+3}{2} = 2$ $\checkmark \text{ answer}$ <div>(2)</div>
4.2	<p>The roots of f are -2 and 6/die wortels van f is -2 en 6</p> $y = a(x-x_1)(x-x_2)$ $y = a(x+2)(x-6)$ $15 = a(1+2)(1-6)$ $15 = -15a$ $\therefore a = -1$ $y = -1(x^2 - 4x - 12)$ $y = -x^2 + 4x + 12$	$\checkmark y = a(x+2)(x-6)$ $\checkmark \text{ subst } (1;15)$ $\checkmark a = -1$ $\checkmark \text{ answer}$ <div>(4)</div>
4.3	$f(2) = -(2)^2 + 4(2) + 12$ $f(2) = 16$ $\therefore TP(2;16)$ <p>Range/waardeversameling g: $y \leq 12$</p> <p>or/of</p> $y \in (-\infty; 12]$	$\checkmark f(2) = 16$ $\checkmark \text{ critical value}$ $\checkmark \text{ notation}$ <div>(3)</div>
		<div>[9]</div>

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

QUESTION 5

5.1	$p = -1$	✓ $p = -1$ (1)
5.2	$y = \frac{a}{x-1}$ $-3 = \frac{a}{0-1}$ $a = 3$ $y = x^2 + bx - 3$ $0 = (1)^2 + (1)b - 3$ $b = 2$	✓ coordinates D(0 ; -3) ✓ substitute (0 ; -3) ✓ substitute (1 ; 0) (3)
5.3	$y = x^2 + 2x - 3$ axis of sym: $x = \frac{-b}{2a}$ $x = \frac{-2}{2(1)}$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4) OR/OF $\frac{dy}{dx} = 0$ $2x + 2 = 0$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4)	✓ substitution ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4) OR/OF ✓ derivative ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4)
5.4	$y \in [-4; \infty)$ or $y \geq -4$	✓ -4 ✓ answer (2)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

5.5.1	$(-5; -8)$	✓ $x = -5$ ✓ $y = -8$ (2)
5.5.2	$y \geq -8$ or $[-8; \infty)$	✓ answer (1)
5.5.3	$m = -5$ $n = g(-5)$ $= \frac{1}{2}(-5) + \frac{9}{2}$ $= 2$	✓ $m = -5$ ✓ substitution ✓ $n = 2$ (3)

QUESTION 6

6.1	$x \in \mathbb{R}; x \neq 0$ OR $x \in (-\infty; 0) \cup (0; \infty)$	✓ $x \in \mathbb{R}$ ✓ $x \neq 0$ (2)
6.2	$y = x + 8$ $7 = m + 8$ $m = -1$	✓ substitution ✓ answer (2)
6.3	$n = 8$	✓ answer (1)
6.4	$p = 1; q = 8$	✓ $p = 1$ ✓ $q = 8$ (2)
6.5	$f(x) = -2x^2 - 4x + 6$ $0 = -2x^2 - 4x + 6$ $0 = x^2 + 2x - 3$ $0 = (x + 3)(x - 1)$ $x = -3$ or $x = 1$	✓ $f(x) = 0$ ✓ factors/formula ✓ both answers (3)
	$E(-1; 7)$ therefore $D(-1; y)$ $y = \frac{-3}{-1} + 8$ $= 11$ $\therefore D(-1; 11)$	✓ substitution ✓ answer (2)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

6.6	$h(x) = -2x + 8$ $-2x + 8 = \frac{8}{x}$ $-2x^2 + 8x = 8$ $-2x^2 + 8x - 8 = 0$ $x^2 - 4x + 4 = 0$ $(x - 2)^2 = 0$ $\therefore x = 2$ $f(2) = \frac{15}{2}$ $h(2) = 4$ $\therefore t = -3\frac{1}{2}$ OR/OF $f(2) = \frac{15}{2}$ <p>Tangent point of contact (2 ; 4)</p> $\therefore 4 = -\frac{1}{2}(2 - 1)^2 + 8 + t$ $4 = \frac{15}{2} + t$ $\therefore t = -3\frac{1}{2}$	$\checkmark x = 2$ $\checkmark f(2) = \frac{15}{2}$ $\checkmark h(2) = 4$ $\checkmark t = -3\frac{1}{2}$ OR/OF $\checkmark x = 2$ $\checkmark f(2) = \frac{15}{2}$ $\checkmark 4$ $\checkmark t = -3\frac{1}{2}$ <div style="text-align: right;">(4)</div> <div style="text-align: right;">(4)</div>
		[18]

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

Nov 2022

QUESTION 7

7.1	$(1 ; 8)$	✓ ✓ $(1 ; 8)$ (2)
7.2	$y = -\frac{1}{2}(0-1)^2 + 8$ $= 7\frac{1}{2}$	✓ substitution of 0 ✓ answer (2)
7.3	$8 = \frac{d}{1}$ $\therefore d = 8$	✓ substitution $(1 ; 8)$ (1)
7.4	$y \in R ; y \neq 0$	✓ $y \in R ; y \neq 0$ (1)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

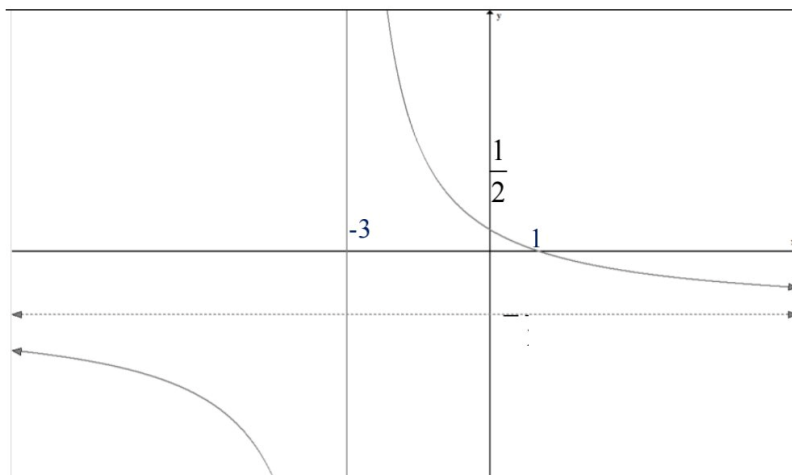
Gauteng trial 2023

QUESTION 8

8.1	$x \neq -3$	✓ answer	(1)
8.2	$y \neq 2$	✓ answer	(1)
8.3	6 units	✓ answer	
8.3.1	NOTE: Accept 6 units right, but not 6 units left.		(1)
8.3.2	3,5 units	✓ answer	
	NOTE: Accept 3,5 units upwards, but not 3,5 units downwards.		(1)
8.4	$x = -3$ $y = -\frac{3}{2}$ NOTE: Do not accept any equating in terms of p and q.	✓ answer ✓ answer	(2)
8.5	$0 = \frac{6}{x+3} - 1,5$ $\frac{3}{2} = \frac{6}{x+3}$ $3(x+3) = 6(2)$ $3x+9 = 12$ $3x = 3$ $x = 1$ (1;0) NOTE: The answer does NOT need to be given in coordinate form.	✓ answer	(1)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

8.6



- ✓ asymptotes
- ✓ intercepts
- ✓ shape

(3)

8.7

$$y = -x + k$$

$$\therefore -\frac{3}{2} = -(-3) + k$$

$$-\frac{3}{2} = 3 + k$$

$$\therefore k = -\frac{9}{2}$$

- ✓ substitute point

- ✓ answer

NOTE: Award FULL marks for Answer only

(2)

8.8

$$x > -3$$

- ✓ answer

(1)

8.9

$$y = \frac{-6}{x+3} + \frac{3}{2}$$

OR

$$y = \frac{6}{-x-3} + \frac{3}{2}$$

OR

$$y = -\frac{6}{x+3} + \frac{3}{2}$$

- ✓ answer

(1)

[14]

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

Gauteng 2021

QUESTION 10

10.1	<p>y-intercept: make $x = 0$</p> <p>y-afsnit: stel $x = 0$</p> $\therefore y = \frac{2}{0+1} - 3$ $y = -1$ $\therefore (0; -1)$ <p>NOTE: Must be in coordinate form. Answer only: full marks/ LET WEL: Moet in koördinaatvorm wees. Antwoord alleenlik: volpunte</p>	<p>✓ answer/antwoord</p> <p>(1)</p>
10.2	<p>x-intercept: make $y = 0$</p> <p>x-afsnit: stel $y = 0$</p> $0 = \frac{2}{x+1} - 3$ $3 = \frac{2}{x+1}$ $3(x+1) = 2$ $3x+3 = 2$ $3x = -1$ $x = -\frac{1}{3}$ $\left(-\frac{1}{3}; 0\right)$ <p>NOTE: Answer does not need to be in coordinate form. LET WEL: Antwoord hoef nie in koördinaatvorm te wees nie.</p>	<p>✓ valid simplification/ geldige vereenvoudiging (either lines 3, 4 or 5/ óf lyne 3, 4 óf 5)</p> <p>✓ answer/antwoord</p> <p>(2)</p>
10.3		<p>✓ shape/vorm</p> <p>✓ asymptotes/asimptotes</p> <p>✓ x-inter/afsnit. $\left(-\frac{1}{3}; 0\right)$ and/en y-inter/afsnit $(0; -1)$</p> <p>(3)</p>

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

10.4	$y = ax + q$ $-3 = -(-1) + q$ $q = -4$ $\therefore y = -x - 4$ OR/OF $y - y_1 = m(x - x_1)$ $y - (-3) = -1(x - (-1))$ $y + 3 = -(x + 1)$ $y + 3 = -x - 1$ $y = -x - 4$ OR/OF $y = -(x + 1) - 3$ $y = -x - 1 - 3$ $y = -x - 4$	✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord OR/OF ✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord OR/OF ✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord (2)
10.5	C(1 ; 0)	✓ answer/antwoord (1)
10.6	Range/waardeversameling: $y \in R$ OR/OF $y \in (-\infty; \infty)$	✓ answer/antwoord (1)
10.7	$y = \log_a x$ $-1 = \log_a 5$ $a^{-1} = 5$ $a = \frac{1}{5}$	✓ correct log equation to exponential equation/ korrekte log vergelyking na eksponensiële vergelyking ✓ answer/antwoord (2)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

November 2014 question 4

QUESTION 11

11.1	$p = 1$ $q = 1$	$\checkmark p$ value /waarde $\checkmark q$ value /waarde (2)
11.2	$0 = \frac{2}{x+1} + 1$ $-x - 1 = 2$ $x = -3$ OR/OF Reflect (0 ; 3) across $y = -x$ to get T(-3 ; 0) $x = -3$ <i>Reflekteer (0 ; 3) om $y = -1$ om T(-3 ; 0) te kry</i> $x = -3$	$\checkmark 0 = \frac{2}{x+1} + 1$ $\checkmark x = -3$ (2) \checkmark reflect across/reflekteer om $y = -x$ $\checkmark x = -3$ (2)
11.3	Shifting g five units to the left shifts (- 1 ; 0) five units to the left. $x = - 6$	\checkmark answer/antwoord (1)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

Gauteng trial 2023

QUESTION 12

12.1	$E\left(\frac{7}{2}, \frac{81}{4}\right)$	✓ answer	(1)
12.2	$f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ $f(1) = -\left(1 - \frac{7}{2}\right)^2 + \frac{81}{4} = 14$ $f(5) = -\left(5 - \frac{7}{2}\right)^2 + \frac{81}{4} = 18$ $\therefore AG = \frac{f(5) - f(1)}{5 - 1}$ $= \frac{18 - 14}{4}$ $\therefore AG = 1$	✓ value of $f(1)$ ✓ value of $f(5)$ ✓ substitution ✓ answer	(4)
12.3	$f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ $\therefore f(x) = -x^2 + 7x + 8$ $\because f(x) = g(x)$ $\therefore -x^2 + 7x + 8 = -3x + 24$ $-x^2 + 10x - 16 = 0$ $\therefore x^2 - 10x + 16 = 0$ $(x - 2)(x - 8) = 0$ $\therefore x = 2 \text{ or } x = 8$ $\therefore x_D = 2$	✓ equating ✓ x-values ✓ answers/selection	(3)
12.4	$ST = f(x) - g(x)$ $ST = -x^2 + 7x + 8 - (-3x + 24)$ $ST = -x^2 + 7x + 8 + 3x - 24$ $ST = -x^2 + 10x - 16$	✓ method ✓ answer	(2)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

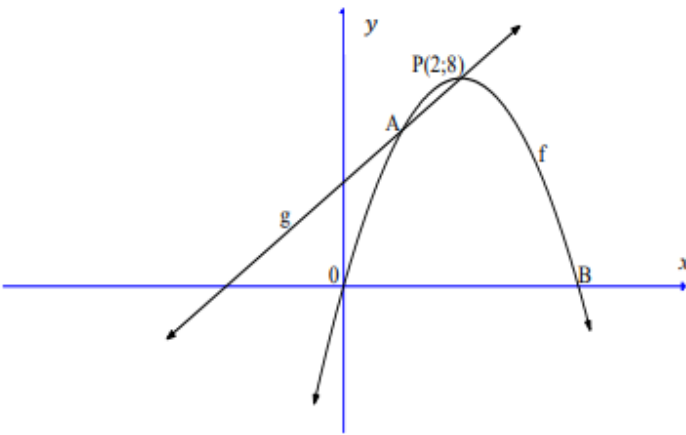
NOV 2019 QUESTION 4

QUESTION 13

13.1	$p = -1$	✓ $p = -1$ (1)
13.2	$y = \frac{a}{x-1}$ $-3 = \frac{a}{0-1}$ $a = 3$ $y = x^2 + bx - 3$ $0 = (1)^2 + (1)b - 3$ $b = 2$	✓ coordinates D(0 ; -3) ✓ substitute (0 ; -3) ✓ substitute (1 ; 0) (3)
13.3	$y = x^2 + 2x - 3$ axis of sym: $x = \frac{-b}{2a}$ $x = \frac{-2}{2(1)}$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4) OR/OF $\frac{dy}{dx} = 0$ $2x + 2 = 0$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4)	✓ substitution ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4) OR/OF ✓ derivative ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4)
13.4	$y \in [-4; \infty)$ or $y \geq -4$	✓ -4 ✓ answer (2)
13.5	$m = \tan 45^\circ = 1$ $y = mx + c$ $-4 = (1)(-1) + c$ $c = -3$ $y = x - 3$	✓ gradient ✓ subs m and $(-1 ; -4)$ ✓ equation (3)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

SEP 2020 FREE STATE QUESTION 4

	QUESTION 14	
14.1		
14.1.1	<p>Substitute/Vervang $P(2; 8)$</p> $y = a(x - 2)^2 + 8$ <p>Substitute/vervang $(0; 0)$</p> $0 = a(0 - 2)^2 + 8$ $a = -2$ $\therefore y = -2(x - 2)^2 + 8$ $= -2(x^2 - 4x + 4) + 8 = -2x^2 + 8x$ <p>ALTERNATIVE/ALTERNATIEWE</p> $y = a(x - 0)(x - 4)$ <p>Substitute/vervang $(2; 8)$</p> $8 = a(2 - 0)(2 - 4)$ $a = -2$ $\therefore y = -2x(x - 4) = -2x^2 + 8x$	<p>✓ substitute/vervang P</p> <p>✓ substitute/vervang $(0; 0)$</p> <p>✓ value of a/ waarde van a</p> <p>✓ simplify/vereenvoudig</p> <p style="text-align: right;">(4)</p> <p>✓ x-intercepts/x-afsnitte</p> <p>✓ substitute P/vervang P</p> <p>✓ value of a/waarde van a</p> <p>✓ simplify/vereenvoudig</p> <p style="text-align: right;">(4)</p>

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

14.1.2	$B(4;0)$ $2x + 4 = -2x^2 + 8x$ $x^2 - 3x + 2 = 0$ $(x-1)(x-2) = 0$ $\therefore A(1;6)$	✓ coordinates B/ <i>koördinate B</i> ✓ equating/ <i>stel gelyk</i> ✓ coordinates A/ <i>koördinate A</i> (3)
14.1.3	$f'(x)g(x) \leq 0$ $\therefore x \in (-\infty; -2] \cup x \in [2; \infty)$ OR/OF $y \leq -2$ or/of $y \geq 2$	✓ values/ <i>waardes</i> ✓ notation/ <i>notasie</i> (2)
14.1.4.	Turning point (1; 7)/ <i>Draaipunt (1; 7)</i> $y \leq 7$ OR/OF $y \in (-\infty; 7]$	✓ (1; 7) ✓ answer/ <i>antwoord</i> Answer only/slegs <i>antwoord 2/2</i> (2)
14.1.5	$f'(x) = -4x + 8$ $\therefore -4x + 8 = 2 \quad x = \frac{3}{2}$	✓ derivative/ <i>afgeleide</i> ✓ 2 ✓ answer/ <i>antwoord</i> (3)
14.2		
14.2.1	$x = 2$ AND/EN $y = 1$	✓✓ (2)
14.2.2	$m = -1$ AND/EN (3; 1) $y - 1 = -1(x - 3)$ OR/OF $1 = -1(3) + c$ $y = -x + 4 \quad c = 4 \therefore y = -x + 4$	✓ $m = -1$ ✓ substitute (3; 1) ✓ answer/ <i>antwoord</i> (3)
		[19]

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

NORTHERN CAPE SEP 2021

QUESTION 15

15.1	$f(x) = -x^2 + 4x + 5$ $x = \frac{-4}{2(-1)} \quad \text{OR} \quad f'(x) = -2x + 4$ $x = 2 \quad \quad \quad 0 = -2x + 4$ $\quad \quad \quad \quad \quad x = 2$ $y = -(2)^2 + 4(2) + 5$ $y = 9$ $B(2;9)$	✓ substitution / derivative ✓ x-value ✓ $y = 8$ (3)
15.2	$p = -2 \text{ and } q = 9$	✓ $p = -2$ ✓ $q = 9$ (2)
15.3	The graph of f reflects over the x -axis and shifts 10 units up to form the graph of $t(x) = -f(x) + 10$. The roots will be non-real. OR $t(x) = -f(x) + 10$	✓✓ answer (2)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

Nov 2015 Question 6

QUESTION 16

16.1	$T(0;18)$	✓ (0;18) (1)
16.2	$-2x^2 + 18 = 0$ $(x-3)(x+3) = 0$ $Q(3;0)$ OR/OF $-2x^2 + 18 = 0$ $x^2 = 9$ $Q(3;0)$	✓ $y = 0$ ✓ factors ✓ $x = 3$ (3) ✓ $y = 0$ ✓ $x^2 = 9$ ✓ $x = 3$ (3)
16.3	<i>x</i> -coordinate of S is 4,5/ <i>x</i> -koördinaat van S is 4,5 By symmetry about the line $x = 4,5$ /Deur simmetrie om die lyn $x = 4,5$: $R = (4,5 + 4,5 - 3; 0) = (6; 0)$	✓ $x = 6$ ✓ $y = 0$ (2)

|
Nov 2019

QUESTION 17

17.1	$f(x) = k^x$ $16 = k^4$ $k = 2$	✓ substitution (4 ; 16) ✓ answer (2)
17.2	$f : y = 2^x$ $f^{-1} : x = 2^y$ $y = \log_2 x$	✓ $x = 2^y$ ✓ $y = \log_2 x$ (2)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

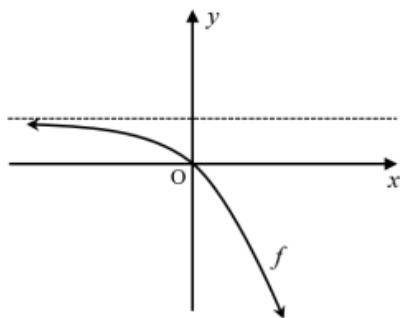
Nov 2020

QUESTION 18

18.1	$A(0; 1)$	✓ answer (1)
18.2	$9 = 3^{-x}$ $3^2 = 3^{-x}$ $x = -2$ $B(-2; 9)$	✓ equating ✓ $3^2 = 3^{-x}$ ✓ $x = -2$ (3)
18.3	$x \in (0; \infty)$ or $x > 0$	✓✓ answer (2)

EASTERN CAPE SEP 2022

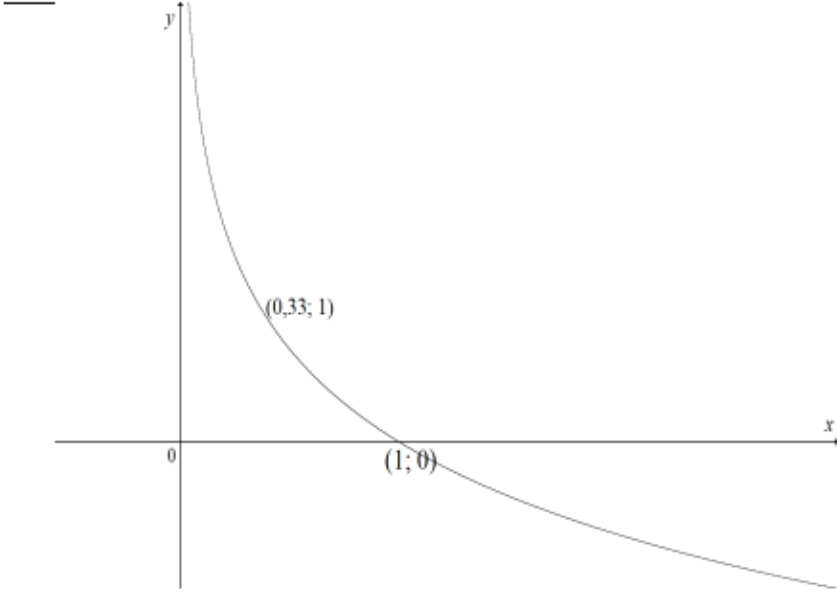
QUESTION 19

19.1		✓ asymptote / asimpotoot ✓ intercept / afsnit ✓ shape / vorm (3)
19.2	$y < 1, y \in \mathbb{R}$ OR/OF $y \in (-\infty; 1)$	✓✓ $y < 1$ Accuracy / Akkuraatheid (2) OR/OF ✓✓ $y \in (-\infty; 1)$ Accuracy/Akkuraatheid (2)
19.3	$g(x) = -(-3^x + 1)$ $= 3^x - 1$ \therefore A s y m p t o t e / A s i m p t o o t : $y = -1$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Answer only – Full Marks Slegs antwoord – Volpunte </div> ✓ $3^x - 1$ ✓ answer / antwoord (2)
19.4	$h(x) = 3^x$ $x = 3^y$ $\therefore y = \log_3 x$	✓ $h(x) = 3^x$ ✓ $x = 3^y$ ✓ answer / antwoord (3)
		[10]

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

FREE STATE SEP 2022

QUESTION 20

20.1	$y = \left(\frac{1}{3}\right)^x$ $k = \left(3^{-1}\right)^{-2}$ $k = 9$	✓ subst $A(-2; k)$ ✓ answer (2)
20.2	$g^{-1}: x = \left(\frac{1}{3}\right)^y$ $y = \log_{\frac{1}{3}} x \text{ or/of } y = -\log_3 x$	✓ swop x and y ✓ answer (2)
20.3		✓ shape ✓ x intercepts ✓ any point (3)
20.4	$-2 < x \leq 7$	✓✓ answer (2)
		[9]

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

NOVEMBER 2021 QUESTION 6

QUESTION 21

21.1	$f(x) = \log_4 x$ $2 = \log_4 k$ $4^2 = k$ $\therefore k = 16$	✓ substitution of $(k; 2)$ ✓ answer (2)
21.2	$-1 = \log_4 x \quad \therefore x = \frac{1}{4}$ $\frac{1}{4} \leq x \leq 16 \quad \text{or/of} \quad x \in \left[\frac{1}{4}; 16 \right]$	✓ $x = \frac{1}{4}$ ✓ answer (2)
21.3	$f(x) = \log_4 x$ $y = \log_4 x$ $x = \log_4 y$ $y = 4^x$	✓ swapping x and y ✓ answer (2)
21.4	$x < 0$ OR/OF $x \in (-\infty; 0)$	✓✓ answer (2) OR/OF ✓✓ answer (2)
		[8]

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

NOV 2019 QUESTION 4
QUESTION 22

22.1	$p = -1$	✓ $p = -1$ (1)
22.2	$y = \frac{a}{x-1}$ $-3 = \frac{a}{0-1}$ $a = 3$ $y = x^2 + bx - 3$ $0 = (1)^2 + (1)b - 3$ $b = 2$	✓ coordinates D(0 ; -3) ✓ substitute (0 ; -3) ✓ substitute (1 ; 0) (3)
22.3	$y = x^2 + 2x - 3$ axis of sym: $x = \frac{-b}{2a}$ $x = \frac{-2}{2(1)}$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4) OR/OF $\frac{dy}{dx} = 0$ $2x + 2 = 0$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4)	✓ substitution ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4) OR/OF ✓ derivative ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4)
22.4	$y \in [-4; \infty)$ or $y \geq -4$	✓ -4 ✓ answer (2)
22.5	$m = \tan 45^\circ = 1$ $y = mx + c$ $-4 = (1)(-1) + c$ $c = -3$ $y = x - 3$	✓ gradient ✓ subs m and $(-1 ; -4)$ ✓ equation (3)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

SEP 2020 QUESTION 5

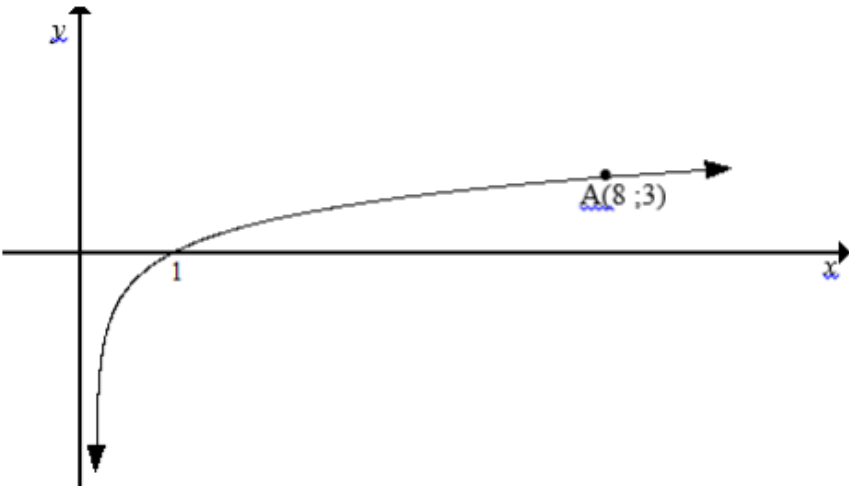
QUESTION 23

23.1	$f(x) = 2^{-x}$ $\therefore g : x = 2^{-y}$ $-y = \log_2 x$ $y = -\log_2 x$ OR/OR $y = \log_2 x^{-1}$ OR/OR $y = \log_{\frac{1}{2}} x$	✓ interchange x and y /ruil x en y om ✓ answer/antwoord (2) Answer only full marks/slegs antwoord volpunte
23.2	Yes, x values do not repeat/Ja, x waardes herhaal nie OR/OR Yes, every x value has unique y value/Ja, elke x waarde het 'n unieke y waarde	✓ yes/ja ✓ valid reason/geldige rede (2)
23.3		f : ✓ shape/vorm ✓ intercept/afsnit g : ✓ shape/vorm ✓ intercept/afsnit (4)
23.4	$h(x) = 2^{-x+1} - 2$ $= 2^{-x} \cdot 2 - 2$ $= 2\left(\frac{1}{2}\right)^x - 2$ OR/OR $y = \frac{2}{2^x} - 2$	✓ correct translation indicated/korrekte translasie aangedui ✓ answer with POSITIVE exponent/antwoord met positiewe eksponent (2)
		[10]

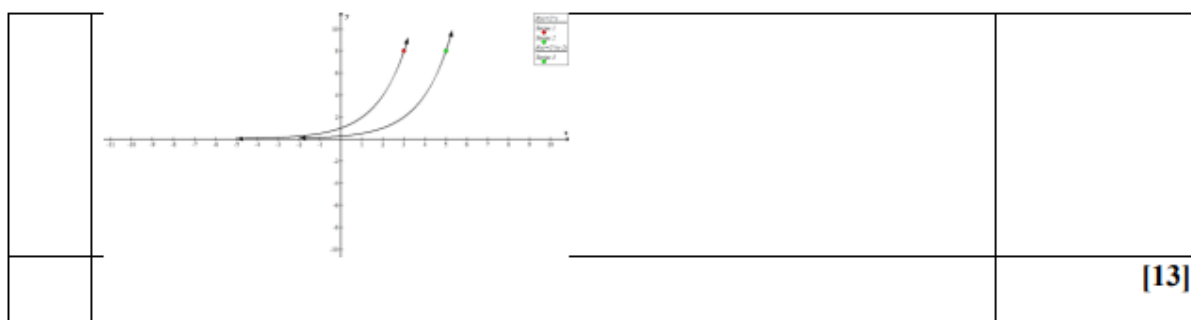
FUNCTIONS LEVEL 1 & 2 SOLUTIONS

NORTHERN CAPE SEP 2021 QUESTION 5

QUESTION 24

24.1	$f(x) = b^x$ $8 = b^x$ $2^3 = b^3$ $b = 2$ $f(x) = 2^x$		✓ substitution ✓ answer (2)
24.2	$y = 2^x$ $x = 2^y$ $y = \log_2 x$	<div>Answer only full marks</div>	✓ $x = 2^y$ ✓ $y = \log_2 x$ (2)
24.3			✓ shape ✓ asymptote ✓ point A(8;3) OR (1;0) (3)
24.4	$\log_2 x < 4$ $x < 2^4$ $0 < x < 16$	<div>Answer only full marks</div>	✓ 16 ✓ ✓ answer (3)
24.5	$h(x) = \frac{1}{4} f(x)$ $= 2^{-2}(2^x)$ $= 2^{x-2}$	x	✓ 2^{x-2} ✓ 2nits ✓ right (3)
The graph shifted 2 units to the right			

FUNCTIONS LEVEL 1 & 2 SOLUTIONS



November 2014 question 4

QUESTION 25

25.1	$p = 1$ $q = 1$	✓ p value /waarde ✓ q value /waarde (2)
25.2	$0 = \frac{2}{x+1} + 1$ $-x - 1 = 2$ $x = -3$ OR/OF Reflect (0 ; 3) across $y = -x$ to get T(-3 ; 0) $x = -3$ Reflekteer (0 ; 3) om $y = -1$ om T(-3 ; 0) te kry $x = -3$	✓ $0 = \frac{2}{x+1} + 1$ ✓ $x = -3$ (2) ✓ reflect across/reflekteer om $y = -x$ ✓ $x = -3$ (2)
25.3	Shifting g five units to the left shifts (-1 ; 0) five units to the left. $x = -6$	✓ answer/antwoord (1)

FUNCTIONS LEVEL 1 & 2 SOLUTIONS

November 2014 Question 5

QUESTION 26

26.1	$y = \log_a x$ $-1 = \log_a \frac{1}{3}$ $a^{-1} = \frac{1}{3}$ $a = \left(\frac{1}{3}\right)^{-1}$ $\therefore a = 3$	\checkmark subst. $\left(\frac{1}{3}; -1\right)$ $\checkmark a^{-1} = \frac{1}{3}$ or $a = \left(\frac{1}{3}\right)^{-1}$ (2)
26.2	$h: x = \log_3 y$ $\therefore y = 3^x$	\checkmark swop x and y /ruil x en y \checkmark answer/antwoord (2)
26.3	$g(x) = -\log_3 x$ OR/OF $g(x) = \log_3 \frac{1}{x}$ OR/OF $g(x) = \log_{\frac{1}{3}} x$ OR/OF $x = 3^{-y}$ OR/OF $x = \left(\frac{1}{3}\right)^y$	\checkmark answer/antwoord (1) \checkmark answer/antwoord (1) \checkmark answer/antwoord (1) \checkmark answer/antwoord (1) \checkmark answer/antwoord (1)
26.4	OR/OF $(0; \infty)$	\checkmark answer/antwoord (1) \checkmark answer/antwoord (1)

QUESTION 1

1.1	$-1 < x < 7$	✓✓ answer / antwoord (2)
1.2	$g(x) - f(x)$ $= 2x - 4 - (x^2 - 4x - 11)$ $= -x^2 + 6x + 7$ For maximum / Vir maksimum: $-2x + 6 = 0$ $\therefore x = 3$	✓ difference / verskil ✓ derivative / afgeleide ✓ equating derivative to 0 stel afgeleide = 0 ✓ answer / antwoord (4)

QUESTION 2

2.1	$f(x) \cdot g(x) \leq 0$ $\therefore x \leq -2$ or/of $-1 \leq x \leq 6$ NOTE: Accuracy marks. Candidate may write answers as separate inequalities. LET WEL: Akkuraatheid punte. Kandidaat mag die antwoord skryf as aparte ongelykhede.	✓ answer/antwoord ✓ answer/antwoord (2)
2.2	$0 < x < 4$	✓ answer/antwoord (1)
2.3	$y = -x - 1$ x - int/afsnit: $0 = -x - 1$ $x = -1$ y - int/afsnit: $y = 0 - 1$ $y = -1$ NOTE: Mark the sketch combined with Question 5.4. If candidate does NOT sketch the line but calculates x - and y -intercepts, award 1 mark. LET WEL: Merk die gekombineerde skets by Vraag 5.4. Indien kandidaat NIE die reguitlyn geskets het nie, maar wel die x - en y -afsnitte bereken het, ken 1 punt toe.	✓ x -intercept/ x -afsnit ✓ y -intercept/ y -afsnit (2)
2.4	$f(x) \cdot g(x) \leq 0$ $\therefore x \leq -2$ or/of $-1 \leq x \leq 6$ NOTE: Accuracy marks. Candidate may write answers as separate inequalities. LET WEL: Akkuraatheid punte. Kandidaat mag die antwoord skryf as aparte ongelykhede.	✓ answer/antwoord ✓ answer/antwoord (2)
[17]		

QUESTION 3

3.1	$p = -1$	✓ $p = -1$ (1)
3.2	$y = \frac{a}{x-1}$ $-3 = \frac{a}{0-1}$ $a = 3$ $y = x^2 + bx - 3$ $0 = (1)^2 + (1)b - 3$ $b = 2$	✓ coordinates D(0 ; -3) ✓ substitute (0 ; -3) ✓ substitute (1 ; 0) (3)
3.3	$y = x^2 + 2x - 3$ axis of sym: $x = \frac{-b}{2a}$ $x = \frac{-2}{2(1)}$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4) OR/OF $\frac{dy}{dx} = 0$ $2x + 2 = 0$ $x = -1$ $y = (-1)^2 + 2(-1) - 3 = -4$ C(-1; -4)	✓ substitution ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4) OR/OF ✓ derivative ✓ $x = -1$ ✓ substitution ✓ $y = -4$ (4)
3.4	$y \in [-4; \infty)$ or $y \geq -4$	✓ -4 ✓ answer (2)
3.5	$m = \tan 45^\circ = 1$ $y = mx + c$ $-4 = (1)(-1) + c$ $c = -3$ $y = x - 3$	✓ gradient ✓ subs m and $(-1 ; -4)$ ✓ equation (3)
3.6	No, the line passes through C and D OR/OF No, a tangent through turning point C will have a gradient of 0	✓ No ✓ reason (2) OR/OF ✓ No ✓ reason (2)

3.7	$f(m-x) = f[-(x-m)]$ f is reflected in the y -axis and translated 1 unit to the left and 4 units upwards. Therefore: $m = -1$ $q = 4$ OR/OF Substitute $x = 0$ and $q = 4$ for one x -intercept $h(x) = (m-x)^2 + 2(m-x) - 3 + q$ $h(0) = (m-0)^2 + 2(m-0) - 3 + 4$ $0 = m^2 + 2m + 1$ $0 = (m+1)^2$ $m = -1$ $q = 4$	✓✓ value of m ✓✓ value of q (4) OR/OF ✓✓ value of m ✓✓ value of q (4)
		[19]

QUESTION 4

4.1	$x \in \mathbb{R}; x \neq 0$ OR $x \in (-\infty; 0) \cup (0; \infty)$	$\checkmark x \in \mathbb{R} \quad \checkmark x \neq 0$ (2) $\checkmark (-\infty; 0) \quad \checkmark (0; \infty)$ (2)
4.2	$y = x + 8$ $7 = m + 8$ $m = -1$	\checkmark substitution \checkmark answer (2)
4.3	$n = 8$	\checkmark answer (1)
4.4	$p = 1; q = 8$	$\checkmark p = 1 \quad \checkmark q = 8$ (2)
4.5	$f(x) = -2x^2 - 4x + 6$ $0 = -2x^2 - 4x + 6$ $0 = x^2 + 2x - 3$ $0 = (x + 3)(x - 1)$ $x = -3 \quad \text{or} \quad x = 1$	$\checkmark f(x) = 0$ \checkmark factors/formula \checkmark both answers (3)
4.6	$E(-1; 7)$ therefore $D(-1; y)$ $y = \frac{-3}{-1} + 8$ $= 11$ $\therefore D(-1; 11)$	\checkmark substitution \checkmark answer (2)

4.7	$g(x) = -\frac{3}{x} + 8$ $= -3x^{-1} + 8$ $g'(x) = 3x^{-2}$ $= \frac{3}{x^2}$ $g'(-1) = \frac{3}{(-1)^2}$ $= 3$ $y - y_1 = m(x - x_1)$ $y - 11 = 3(x + 1)$ $y = 3x + 14$	$\checkmark -3x^{-1}$ $\checkmark g'(x) = 3x^{-2}$ \checkmark substitution \checkmark substitution \checkmark answer (5)
4.8	$y = \frac{3}{x-4} + 8$	$\checkmark 3 \quad \checkmark x - 4$ $\checkmark y = \frac{3}{x-4} + 8$ (3)
4.9	g shifts 4 units left $\therefore D'(-5 ; 11)$ with an asymptote at $y = 8$ $\therefore g$ must shift more than 8 units down, but less than 11 units down. $8 < -k < 11$ $-11 < k < -8$	$\checkmark D'(-5 ; 11)$ $\checkmark 8$ \checkmark notation (3) [23]

QUESTION 5

5.1	$\begin{pmatrix} -4 \\ 6 \end{pmatrix}$	$\checkmark B(-4 ; 0)$ $\checkmark D(6 ; 0)$ (2)
5.2	$f(x) = x^2 - 2x - 24$ $x_{tp} = \frac{-b}{2a}$ OR/OF $2x - 2 = 0$ OR/OF $x = \frac{-4+6}{2}$ $x = \frac{-(-2)}{2(1)}$ $\therefore x_{tp} = 1$ $y_{tp} = f(1)$ $= 1^2 - 2(1) - 24$ $= -25$ $C(1 ; -25)$	$\checkmark x_{tp} = 1$ $\checkmark y_{tp} = -25$ (2)
5.3	$y \geq -25$ OR/OF $y \in [-25 ; \infty)$	\checkmark answer (1) OR/OF \checkmark answer (1)
5.4.1	$_{AE} = \tan 14,04^\circ = 0,25 = \frac{1}{4}$	\checkmark answer (1)
5.4.2	$_{ang} = -4$ $f'(x) = 2x - 2$ $2x - 2 = -4$ $x_T = -1$ $y_T = -21$	$\checkmark m_{tang} = -4$ $\checkmark f'(x) = 2x - 2$ \checkmark equating $\checkmark x_T = -1$ $\checkmark y_T = -21$ (5)
5.4.5	$_{line} = \frac{1}{4}$ $y + 9 = \frac{1}{4}(x + 3)$ OR/OF $-9 = \frac{1}{4}(-3) + c$ $y + 9 = \frac{1}{4}x + \frac{3}{4}$ $c = -\frac{33}{4} = -8,25$ $y = \frac{1}{4}x - \frac{33}{4}$ OR/OF $y = 0,25x - 8,25$ $x^2 - 2x - 24 = \frac{1}{4}x - \frac{33}{4}$ $4x^2 - 8x - 96 = x - 33$ $4x^2 - 9x - 63 = 0$ $(4x - 21)(x + 3) = 0$ $\therefore x = \frac{21}{4} = 5,25$ or $x \neq -3$	$\checkmark m_{line} = \frac{1}{4}$ \checkmark substitution m and $K(-3 ; -9)$ $\checkmark y = \frac{1}{4}x - \frac{33}{4}$ \checkmark equating \checkmark standard form \checkmark answer with selection (6)
		[17]

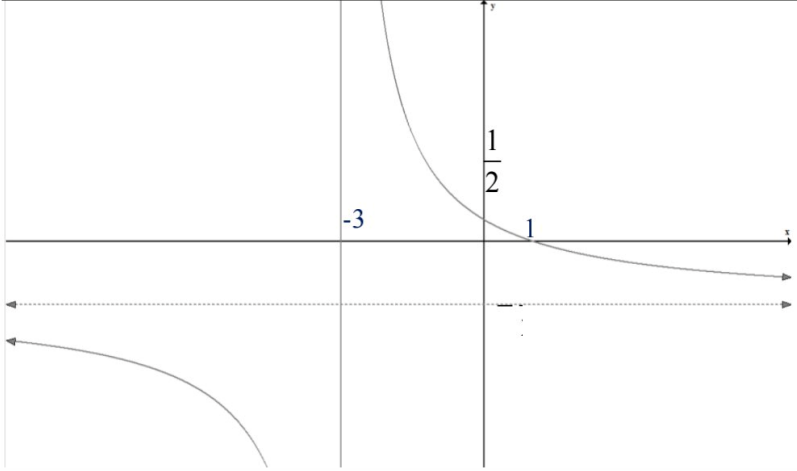
QUESTION 6

6.1	$(1; 8)$	✓✓ $(1; 8)$ (2)
6.2	$y = -\frac{1}{2}(0-1)^2 + 8$ $= 7\frac{1}{2}$	✓ substitution of 0 ✓ answer (2)
6.3	$8 = \frac{d}{1}$ $\therefore d = 8$	✓ substitution $(1; 8)$ (1)
6.4	$y \in R; y \neq 0$	✓ $y \in R; y \neq 0$ (1)
6.5	$-3 \leq x < 0$ or $x \geq 5$ OR/OF $[-3; 0) \cup [5; \infty)$	✓✓ $-3 \leq x < 0$ ✓ $x \geq 5$ (3)
6.6	$-2x + k = \frac{8}{x}$ $-2x^2 + kx - 8 = 0$ $\Delta = (k)^2 - 4(-2)(-8)$ $k^2 - 64 < 0$ $CV: k = 8; k = -8$ $\therefore -8 < k < 8$ OR/OF $f'(x) = h'(x)$ $-\frac{8}{x^2} = -2$ $-8 = -2x^2$ $x = \pm 2$ $y = \pm 4 \therefore B(2; 4) \text{ and } A(-2; -4)$ For tangents: $h(x) = -2x + k$ or $h(x) = -2x + k$ $4 = -2(2) + k$ $-4 = -2(-2) + k$ $k = 8$ $k = -8$ $\therefore -8 < k < 8$	✓ $-2x + k = \frac{8}{x}$ ✓ standard form ✓ $\Delta = k^2 - 64$ ✓ $\Delta < 0$ ✓ $\therefore -8 < k < 8$ (5) OR/OF ✓ $-\frac{8}{x^2} = -2$ ✓ $x = \pm 2$ ✓ $y = \pm 4$ ✓ $\therefore -8 < k < 8$ (5)

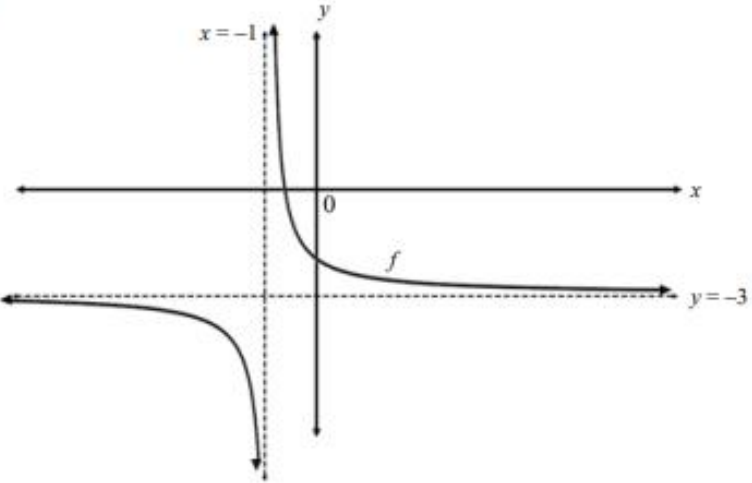
6.7	$h(x) = -2x + 8$ $-2x + 8 = \frac{8}{x}$ $-2x^2 + 8x = 8$ $-2x^2 + 8x - 8 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $\therefore x = 2$ $f(2) = \frac{15}{2}$ $h(2) = 4$ $\therefore t = -3\frac{1}{2}$ OR/OF $f(2) = \frac{15}{2}$ <p>Tangent point of contact (2 ; 4)</p> $\therefore 4 = -\frac{1}{2}(2-1)^2 + 8 + t$ $4 = \frac{15}{2} + t$ $\therefore t = -3\frac{1}{2}$	$\checkmark x = 2$ $\checkmark f(2) = \frac{15}{2}$ $\checkmark h(2) = 4$ $\checkmark t = -3\frac{1}{2}$ <p style="text-align: right;">(4)</p> OR/OF $\checkmark x = 2$ $\checkmark f(2) = \frac{15}{2}$ $\checkmark 4$ $\checkmark t = -3\frac{1}{2}$ <p style="text-align: right;">(4)</p>
		[18]

QUESTION 7

7.1	$x \neq -3$	✓ answer	(1)
7.2	$y \neq 2$	✓ answer	(1)
7.3	7.3.1 6 units	✓ answer	
	NOTE: Accept 6 units right, but not 6 units left.		(1)
7.3.2	3,5 units	✓ answer	
	NOTE: Accept 3,5 units upwards, but not 3,5 units downwards.		(1)
7.4	$x = -3$	✓ answer	
	$y = -\frac{3}{2}$	✓ answer	
	NOTE: Do not accept any equating in terms of p and q.		(2)
7.5	$y = \frac{6}{x+3} - 1,5$ $\frac{3}{2} = \frac{6}{x+3}$ $3(x+3) = 6(2)$ $3x+9 = 12$ $3x = 3$ $x = 1$ $(1;0)$ NOTE: The answer does NOT need to be given in coordinate form.	✓ answer	(1)

7.6		<ul style="list-style-type: none"> ✓ asymptotes ✓ intercepts ✓ shape 	(3)
7.7	$y = -x + k$ $\therefore -\frac{3}{2} = -(-3) + k$ $-\frac{3}{2} = 3 + k$ $\therefore k = -\frac{9}{2}$ <p>NOTE: Award FULL marks for Answer only</p>	<ul style="list-style-type: none"> ✓ substitute point ✓ answer 	(2)
7.8	$x > -3$	✓ answer	(1)
7.9	$y = \frac{-6}{x+3} + \frac{3}{2}$ <p>OR</p> $y = \frac{6}{-x-3} + \frac{3}{2}$ <p>OR</p> $y = -\frac{6}{x+3} + \frac{3}{2}$	✓ answer	(1)
[14]			

QUESTION 8

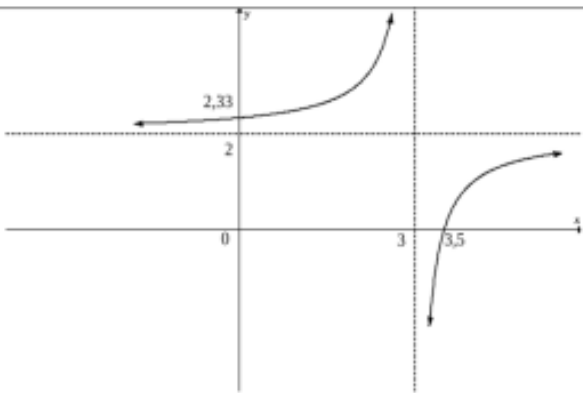
8.1	<p>y-intercept: make $x = 0$</p> <p>y-afsnit: stel $x = 0$</p> $\therefore y = \frac{2}{0+1} - 3$ $y = -1$ $\therefore (0; -1)$ <p>NOTE: Must be in coordinate form. Answer only: full marks/ LET WEL: Moet in koördinaatvorm wees. Antwoord alleenlik: volpunte</p>	<p>✓ answer/antwoord</p> <p>(1)</p>
8.2	<p>x-intercept: make $y = 0$/</p> <p>x-afsnit: stel $y = 0$</p> $0 = \frac{2}{x+1} - 3$ $3 = \frac{2}{x+1}$ $3(x+1) = 2$ $3x+3 = 2$ $3x = -1$ $x = -\frac{1}{3}$ $\left(-\frac{1}{3}; 0\right)$ <p>NOTE: Answer does not need to be in coordinate form. LET WEL: Antwoord hoef nie in koördinaatvorm te wees nie.</p>	<p>✓ valid simplification/ geldige vereenvoudiging (either lines 3, 4 or 5/ óf lyne 3, 4 óf 5)</p> <p>✓ answer/antwoord</p> <p>(2)</p>
8.3		<p>✓ shape/vorm</p> <p>✓ asymptotes/asimptotes</p> <p>✓ x-inter/afsnit. $\left(-\frac{1}{3}; 0\right)$ and/en y-inter/afsnit $(0; -1)$</p> <p>(3)</p>

8.4	$y = ax + q$ $-3 = -(-1) + q$ $q = -4$ $\therefore y = -x - 4$ OR/OF $y - y_1 = m(x - x_1)$ $y - (-3) = -1(x - (-1))$ $y + 3 = -(x + 1)$ $y + 3 = -x - 1$ $y = -x - 4$ OR/OF $y = -(x + 1) - 3$ $y = -x - 1 - 3$ $y = -x - 4$	✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord OR/OF ✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord OR/OF ✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord (2)
8.2		
8.2.1	C(1 ; 0)	✓ answer/antwoord (1)
8.2.2	Range/waardeversameling: $y \in R$ OR/OF $y \in (-\infty; \infty)$	✓ answer/antwoord (1)
8.2.3	$y = \log_a x$ $-1 = \log_a 5$ $a^{-1} = 5$ $a = \frac{1}{5}$	✓ correct log equation to exponential equation/ korrekte log vergelyking na eksponensiële vergelyking ✓ answer/antwoord (2)

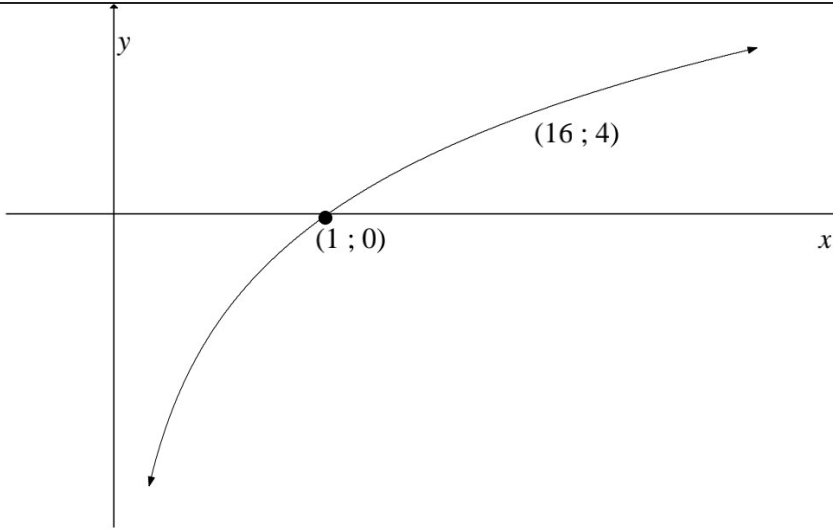
QUESTION 9

9.1	$E\left(\frac{7}{2}; \frac{81}{4}\right)$	✓ answer	(1)
9.2	$f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ $f(1) = -\left(1 - \frac{7}{2}\right)^2 + \frac{81}{4} = 14$ $f(5) = -\left(5 - \frac{7}{2}\right)^2 + \frac{81}{4} = 18$ $\therefore AG = \frac{f(5) - f(1)}{5 - 1}$ $= \frac{18 - 14}{4}$ $\therefore AG = 1$	✓ value of $f(1)$ ✓ value of $f(5)$ ✓ substitution ✓ answer	(4)
9.3	$f(x) = -\left(x - \frac{7}{2}\right)^2 + \frac{81}{4}$ $\therefore f(x) = -x^2 + 7x + 8$ $\therefore f(x) = g(x)$ $\therefore -x^2 + 7x + 8 = -3x + 24$ $-x^2 + 10x - 16 = 0$ $\therefore x^2 - 10x + 16 = 0$ $(x - 2)(x - 8) = 0$ $\therefore x = 2 \text{ or } x = 8$ $\therefore x_D = 2$	✓ equating ✓ x-values ✓ answers/selection	(3)
9.4	$ST = f(x) - g(x)$ $ST = -x^2 + 7x + 8 - (-3x + 24)$ $ST = -x^2 + 7x + 8 + 3x - 24$ $ST = -x^2 + 10x - 16$	✓ method ✓ answer	(2)

QUESTION 10

10.1	$x = 3$ $y = 2$	$\checkmark x = 3$ $\checkmark y = 2$ (2)
10.2	$x \in R, x \neq 3$ OR/OF $x \in (-\infty ; 3) \cup (3 ; \infty)$ OR/OF $x < 3 \text{ or } x > 3$	\checkmark answer (1) OR/OF \checkmark answer (1) OR/OF \checkmark answer (1)
10.3	$0 = \frac{-1}{x-3} + 2$ $-2x + 6 = -1$ $x = \frac{7}{2}$ $x\text{-int: } \left(\frac{7}{2}; 0\right)$	$\checkmark y = 0$ \checkmark answer (2)
10.4	$y\text{-int: } \left(0; \frac{7}{3}\right)$	$\checkmark x = 0$ $\checkmark \frac{7}{3}$ (2)
10.5		\checkmark asymptotes \checkmark intercepts with the axes \checkmark shape (3)
		[10]

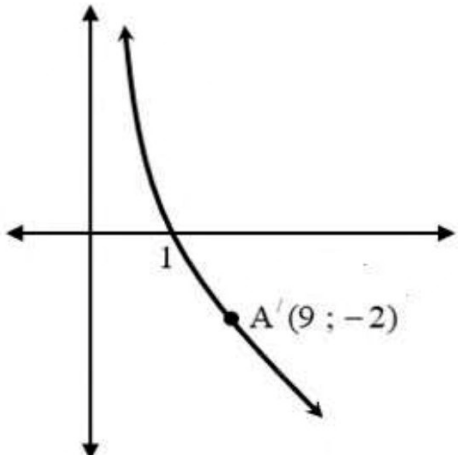
QUESTION 11

11.1	$f(x) = k^x$ $16 = k^4$ $k = 2$	✓ substitution (4 ; 16) ✓ answer (2)
11.2	$f : y = 2^x$ $f^{-1} : x = 2^y$ $y = \log_2 x$	✓ $x = 2^y$ ✓ $y = \log_2 x$ (2)
11.3		✓ asymptote ✓ shape ✓ for any two valid points eg.(16 ; 4) or (2 ; 1) or (4 ; 2) or (1 ; 0) (4)
11.4.1	$x \in (1 ; \infty)$ or $x > 1$	✓ 1 ✓ answer (2)
11.4.2	$0 < x \leq \frac{1}{2}$ or $x \in \left(0; \frac{1}{2}\right]$	✓ $\frac{1}{2}$ ✓ answer (2)

QUESTION 12

12.1	A(0 ; 1)	✓ answer (1)
12.2	$9 = 3^{-x}$ $3^2 = 3^{-x}$ $x = -2$ B(-2 ; 9)	✓ equating ✓ $3^2 = 3^{-x}$ ✓ $x = -2$ (3)
12.3	$x \in (0; \infty)$ or $x > 0$	✓✓ answer (2)
12.4	$h(x) = 27 \cdot 3^{-x}$ $h(x) = 3^{-(x-3)}$ <i>f</i> shifted 3 units to the right	✓ $h(x) = 3^{-(x-3)}$ ✓ 3 units ✓ right (3)
12.5	$\frac{27}{3^x} < 1$ $3^{-x+3} < 1$ $3^x > 27$ or $3^{-x+3} < 3^0$ $3^x > 3^3$ $-x+3 < 0$ $x > 3$ $x > 3$ OR The graph shifts 3 units to the right Thus the y-intercept shift 3 units to the right (3 ; 1) $\therefore x > 3$	✓ $3^x > 27$ or $3^{-x+3} < 3^0$ ✓ $3^x > 3^3$ or $-x+3 < 0$ ✓ $x > 3$ (3) OR ✓ translation ✓ y-intercept ✓ answer (3)
		[12]

QUESTION 13

13.1	$f: y = 2x^2; y \geq 0$ $f^{-1}: x = 2y^2$ $y^2 = \frac{x}{2}$ $y = \pm \sqrt{\frac{x}{2}}; x \geq 0$	<p>✓ swop x and y</p> <p>✓ answer for y ✓ restriction for x</p> <p>(3)</p>
13.2	 <p>$A'(9; -2)$ $\therefore 1 \leq x \leq 9$</p>	<p>✓ $A'(9; -2)$ ✓ 1 ✓ notation</p> <p>(3)</p>
13.3	$f\left(\frac{1}{x}\right) + \frac{1}{f(x)} + [f^{-1}(x)]^2$ $= 2\left(\frac{1}{x}\right)^2 + \frac{1}{2x^2} + \frac{x}{2}$ $= \frac{2}{x^2} + \frac{1}{2x^2} + \frac{x}{2}$ $= \frac{5 + x^3}{2x^2}$	<p>✓ $2\left(\frac{1}{x}\right)^2$ ✓ $\frac{1}{2x^2}$</p> <p>✓ answer</p> <p>(3) [9]</p>

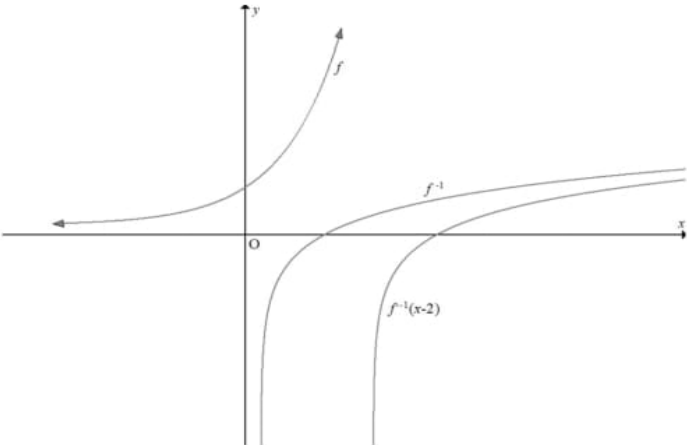
QUESTION 14

14.1	$f(x) = \log_4 x$ $2 = \log_4 k$ $4^2 = k$ $\therefore k = 16$	✓ substitution of $(k ; 2)$ ✓ answer (2)
14.2	$-1 = \log_4 x \therefore x = \frac{1}{4}$ $\frac{1}{4} \leq x \leq 16$ or/of $x \in \left[\frac{1}{4} ; 16 \right]$	✓ $x = \frac{1}{4}$ ✓ answer (2)
14.3	$f(x) = \log_4 x$ $y = \log_4 x$ $x = \log_4 y$ $y = 4^x$	✓ swapping x and y ✓ answer (2)
14.4	$x < 0$ OR/OF	✓✓ answer (2) OR/OF
	$x \in (-\infty ; 0)$	✓✓ answer (2)
		[8]

QUESTION 15

15.1	$y = -4$	✓ $y = -4$ (1)
15.2	x – intercept: $0 = 2^x - 4$ $4 = 2^x$ $x = 2$ $\therefore B(2; 0)$	✓ $y = 0$ ✓ simplification (2)
15.3	$y = 2^0 - 4 = -3$ $\therefore A(0; -3)$ $y = mx + c$ $m = \frac{3}{2}$ $k(x) = \frac{3}{2}x - 3$	✓ -3 ✓ $m = \frac{3}{2}$ ✓ $k(x) = \frac{3}{2}x - 3$ (3)
15.4	$k(1) = \frac{3}{2}(1) - 3 = -\frac{3}{2}$ $f(1) = 2^1 - 4 = -2$ Vertical distance = $-\frac{3}{2} - (-2) = \frac{1}{2}$ units	✓ $-\frac{3}{2}$ ✓ -2 ✓ answer (3)
15.5	$g(x) = f(x) + 4$ $g(x) = 2^x; x \in [-2; 4)$	✓ $g(x) = 2^x$ (1)
15.6	Range of $g: y \in \left[\frac{1}{4}; 16\right)$ Domain of $g^{-1}: x \in \left[\frac{1}{4}; 16\right)$	✓ ✓ $x \in \left[\frac{1}{4}; 16\right)$ (2)
15.7	$g: y = 2^x$ $g^{-1}: x = 2^y$ $g^{-1}(x) = \log_2 x, x \in \left[\frac{1}{4}; 16\right)$	✓ swop x and y ✓ $\log_2 x$ (2)
		[14]

QUESTION 17

17.1	$y = 3^x$ Inverse $x = 3^y$ $\therefore y = \log_3 x$	✓ answer	(1)
17.2	$y = f^{-1}(x) = \log_3 x$ x-intercept: $\log_3 x = 0$ $\therefore x = 3^0$ $\therefore x = 1$ y-intercept: None $y = f^{-1}(x-2) = \log_3(x-2)$ x-intercept: $\log_3(x-2) = 0$ $\therefore x-2 = 3^0$ $\therefore x = 3$ y-intercept: None  NOTE If the candidate does not draw the graph but calculates both x-intercepts, award 2 marks. If the graph crosses the asymptote, DO NOT award the shape mark.	✓ x-intercept of $f^{-1}(x) = \log_3 x$ ✓ shape $f^{-1}(x) = \log_3 x$ ✓ x-intercept of $f^{-1}(x-2) = \log_3(x-2)$ ✓ shape $f^{-1}(x-2) = \log_3(x-2)$	(4)
17.3	$\log_3(x-2) < 1$ $\therefore 2 < x < 5$	✓ critical values ✓ correct notation	(2)
[7]			

QUESTION 18

18.1

y-intercept: make $x = 0$

y-afsnit: stel $x = 0$

$$\therefore y = \frac{2}{0+1} - 3$$

$$y = -1$$

$$\therefore (0; -1)$$

NOTE: Must be in coordinate form.

Answer only: full marks/

LET WEL: Moet in koördinaatvorm wees.

Antwoord alleenlik: volpunte

✓ answer/antwoord

(1)

18.2

x-intercept: make $y = 0$

x-afsnit: stel $y = 0$

$$0 = \frac{2}{x+1} - 3$$

$$3 = \frac{2}{x+1}$$

$$3(x+1) = 2$$

$$3x+3 = 2$$

$$3x = -1$$

$$x = -\frac{1}{3}$$

$$\left(-\frac{1}{3}; 0\right)$$

✓ valid simplification/
geldige vereenvoudiging
(either lines 3, 4 or 5/
óf lyne 3, 4 óf 5)

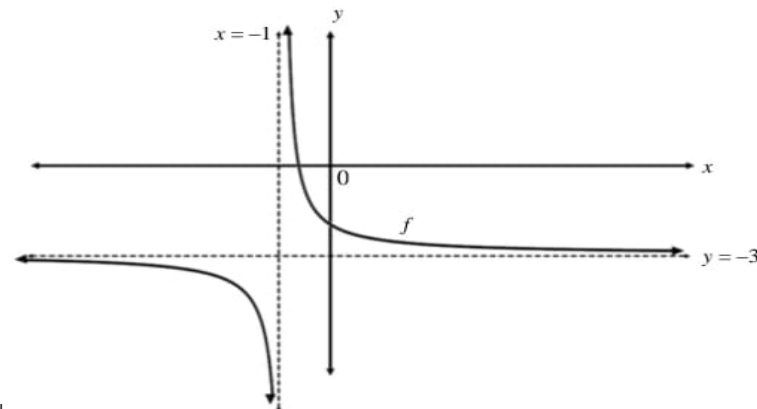
✓ answer/antwoord

(2)

NOTE: Answer does not need to be in coordinate form.

LET WEL: Antwoord hoef nie in koördinaatvorm te wees nie.

18.3



✓ shape/vorm

✓ asymptotes/asimptotes

✓ x-inter/afsnit. $\left(-\frac{1}{3}; 0\right)$
and/en
y-inter/afsnit $(0; -1)$

(3)

QUESTION 19

19.1	$y = ax + q$ $-3 = -(-1) + q$ $q = -4$ $\therefore y = -x - 4$ OR/OF $y - y_1 = m(x - x_1)$ $y - (-3) = -1(x - (-1))$ $y + 3 = -(x + 1)$ $y + 3 = -x - 1$ $y = -x - 4$ OR/OF $y = -(x + 1) - 3$ $y = -x - 1 - 3$ $y = -x - 4$	✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord OR/OF ✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord OR/OF ✓ substitute/vervang $m = -1$ and point/en punt $(-1 ; -3)$ ✓ answer/antwoord (2)
19.2	C(1 ; 0)	✓ answer/antwoord (1)
19.3	Range/waardeversameling: $y \in R$ OR/OF $y \in (-\infty; \infty)$	✓ answer/antwoord (1)
19.4	$y = \log_a x$ $-1 = \log_a 5$ $a^{-1} = 5$ $a = \frac{1}{5}$	✓ correct log equation to exponential equation/ korrekte log vergelyking na eksponensiële vergelyking ✓ answer/antwoord (2)
19.5	$t: y = \log_{\frac{1}{5}} x$ $h: x = \log_{\frac{1}{5}} y$ $y = \left(\frac{1}{5}\right)^x$ OR/OF $y = 5^{-x}$	✓ interchange x and y / ruil x en y om ✓ answer/antwoord (2)

QUESTION 20

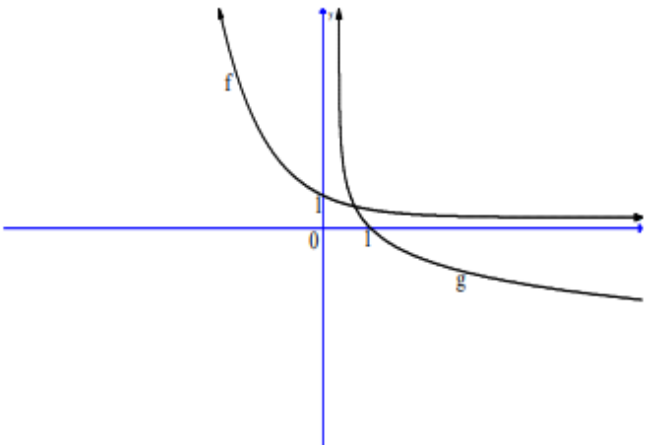
20.1	$x \in R$ OR/OF $x \in (-\infty; \infty)$	✓ answer/antwoord OR/OF ✓ answer/antwoord (1)
20.2	$y = 0$	✓ answer/antwoord (1)
[16]		

22 E/C

20.3	$y = -(x-2) + 2$ $y = -x + 4$ $\therefore k = 4$	$y = -x + k$ $2 = -2 + k$ $\therefore k = 4$ ✓ substitution / vervanging ✓ answer / antwoord (2)
20.4	$y = \frac{8}{(x-5)} + 2$ $y = -\left[\frac{8}{(x-5)} + 2\right]$ $y = -\frac{8}{(x-5)} - 2$	✓ shift 3 units to the right <i>skuif 3 eenhede na regs</i> ✓ reflection in the x-axis <i>refleksie in die x-as</i> ✓ answer / antwoord (3)

20.5	$f(m-x) = f[-(x-m)]$ <i>f</i> is reflected in the y-axis and translated 1 unit to the left and 4 units upwards. Therefore: $m = -1$ $q = 4$ OR/OF Substitute $x = 0$ and $q = 4$ for one x -intercept $h(x) = (m-x)^2 + 2(m-x) - 3 + q$ $h(0) = (m-0)^2 + 2(m-0) - 3 + 4$ $0 = m^2 + 2m + 1$ $0 = (m+1)^2$ $m = -1$ $q = 4$	✓✓ value of m ✓✓ value of q (4) OR/OF ✓✓ value of m ✓✓ value of q (4)
[19]		

QUESTION 21

<p>21.1</p>		<p>f : ✓ shape/vorm ✓ intercept/afsnit g : ✓ shape/vorm ✓ intercept/afsnit</p>
<p>21.2</p>	<p>$h(x) = 2^{-x+1} - 2$</p> <p>$= 2^{-x} \cdot 2 - 2$</p> <p>$= 2\left(\frac{1}{2}\right)^x - 2$ OR/OF $y = \frac{2}{2^x} - 2$</p>	<p>✓ correct translation indicated/korrekte translasiie aangedui ✓ answer with POSITIVE exponent/antwoord met positiewe eksponent</p>

QUESTION 22

22.1	The graph shifted 2 units to the right and 6 units down $\therefore m = -2$ and $n = -6$	✓ $m = -2$ ✓ $n = -6$ (2)
22.2	$y = 8x + k$ $f(x) = -x^2 + 4x + 5$ $f'(x) = -2x + 4$ $8 = -2x + 4$ $4 = -2x$ $x = -2$ $y = -(-2)^2 + 4(-2) + 5$ $y = -7$ $\therefore P(-2; -7)$	✓ derivative ✓ $6 = -2x + 4$ ✓ x-value ✓ y-value (4)
22.3	$2 < x < 3$ or $x \in (2; 3)$	✓ critical value(s) ✓ notation (2)
22.4	$0 = -x^2 + 4x + 5$ $0 = x^2 - 4x - 5$ $(x - 5)(x + 1) = 0$ $x = -1$ or $x = 5$ $\therefore x < -1$ or $x > 5$	✓ standard form=0 ✓ Critical value(s) ✓ $x < -1$ ✓ $x > 5$ (4)
		[19]

QUESTION 23

23.1	$m = \tan 45^\circ = 1$ $y = mx + c$ $-4 = (1)(-1) + c$ $c = -3$ $y = x - 3$	✓ gradient ✓ subs m and $(-1; -4)$ ✓ equation (3)
23.2	No, the line passes through C and D OR/OF No, a tangent through turning point C will have a gradient of 0	✓ No ✓ reason (2) OR/OF ✓ No ✓ reason (2)

QUESTION 24

24.1	$m = \tan 45^\circ = 1$ $y = mx + c$ $-4 = (1)(-1) + c$ $c = -3$ $y = x - 3$	✓ gradient ✓ subs m and $(-1; -4)$ ✓ equation (3)
24.2	No, the line passes through C and D OR/OF No, a tangent through turning point C will have a gradient of 0	✓ No ✓ reason (2) OR/OF ✓ No ✓ reason (2)
24.3	$f(m-x) = f[-(x-m)]$ f is reflected in the y -axis and translated 1 unit to the left and 4 units upwards. Therefore: $m = -1$ $q = 4$ OR/OF Substitute $x = 0$ and $q = 4$ for one x -intercept $h(x) = (m-x)^2 + 2(m-x) - 3 + q$ $h(0) = (m-0)^2 + 2(m-0) - 3 + 4$ $0 = m^2 + 2m + 1$ $0 = (m+1)^2$ $m = -1$ $q = 4$	✓✓ value of m ✓✓ value of q (4) OR/OF ✓✓ value of m ✓✓ value of q (4)

QUESTION 25

25.1	$-1 = \log_4 x \therefore x = \frac{1}{4}$	✓ $x = \frac{1}{4}$	
25.1	$\frac{1}{4} \leq x \leq 16$ or/of $x \in \left[\frac{1}{4}; 16\right]$	✓ answer	(2)
	$f(x) = \log_4 x$ $y = \log_4 x$ $x = \log_4 y$ $y = 4^x$	✓ swapping x and y ✓ answer	(2)
25.2	$x < 0$	✓✓ answer	(2)
	OR/OF	OR/OF	
	$x \in (-\infty; 0)$	✓✓ answer	(2)

QUESTION 26

26.1	$f(x).g(x) \leq 0$ $\therefore x \leq -2$ or/of $-1 \leq x \leq 6$ NOTE: Accuracy marks. Candidate may write answers as separate inequalities. LET WEL: Akkuraatheid punte. Kandidaat mag die antwoord skryf as aparte ongelykhede.	✓ answer/antwoord ✓ answer/antwoord (2)
26.2	$0 < x < 4$	✓ answer/antwoord (1)
26.3	$y = -x - 1$ x – int/afsnit: $0 = -x - 1$ $x = -1$ y – int/afsnit: $y = 0 - 1$ $y = -1$ NOTE: Mark the sketch combined with Question 5.4. If candidate does NOT sketch the line but calculates x- and y-intercepts, award 1 mark. LET WEL: Merk die gekombineerde skets by Vraag 5.4. Indien kandidaat NIE die reguitlyn geskets het nie, maar wel die x- en y-afsnitte bereken het, ken 1 punt toe.	✓ x-intercept/x-afsnit ✓ y-intercept/y-afsnit (2)
26.4	$f(x).g(x) \leq 0$ $\therefore x \leq -2$ or/of $-1 \leq x \leq 6$ NOTE: Accuracy marks. Candidate may write answers as separate inequalities. LET WEL: Akkuraatheid punte. Kandidaat mag die antwoord skryf as aparte ongelykhede.	✓ answer/antwoord ✓ answer/antwoord (2)

[17]

12. COGNITIVE LEVEL 1&2 SOLUTIONS (GRADE 11 REVISION)

12.1 Class Activity/Home Activity A

1.1	$A = P(1+i)^n$ $= 140\,000(1+0,061)^4$ $= R177\,414,69$	$\checkmark 140\,000$ $\checkmark (1+0,061)^4$ $\checkmark \text{answer/antwoord} \quad (3)$
1.2	$1+i_{\text{eff}} = \left(1 + \frac{0,07}{2}\right)^2$ $1+i_{\text{eff}} = (1+0,035)^2$ $i_{\text{eff}} = (1+0,035)^2 - 1$ $= 0,071225$ <p>The effective interest rate/Die <i>effektiewe rentekoers</i> is 7,12% p.a.</p>	$\checkmark \frac{0,07}{2}$ $\checkmark 1+i_{\text{eff}} = (1+0,035)^2$ $\checkmark \text{answer/antwoord} \quad (3)$
1.3	$102\,755,34 = 198\,000 \left(1 - \frac{r}{100}\right)^3$ $\sqrt[3]{\frac{102\,755,34}{198\,000}} = 1 - \frac{r}{100}$ $1 - \frac{r}{100} = 0,8036119818$ $-\frac{r}{100} = -0,1963880182$ $r = 19,64\%$	$\checkmark \text{formula}$ $\checkmark \text{correct substitution into correct formula/korr subst in korr formule}$ $\checkmark n = 3$ $\checkmark \sqrt[3]{\frac{102\,755,34}{198\,000}} = 1 - \frac{r}{100}$ $\checkmark \text{answer/antwoord} \quad (5)$
1.4	$A = P(1-i)^n$ $= R\,25\,000 (1-0,09)^4$ $= R17\,143,74$	$\checkmark A = P(1-i)^n$ $\checkmark \text{substitution/verv.}$ $\checkmark \text{answer/antw.} \quad (3)$
1.5	$1+i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{m}\right)^m$ $1+i_{\text{eff}} = \left(1 + \frac{0,1235}{12}\right)^{12}$ $i_{\text{eff}} = \left(1 + \frac{0,1235}{12}\right)^{12} - 1$ $\therefore \text{Rate} = 0,13073 \times 100$ $= 13,07\%$ <p>The effective interest rate/Die <i>effektiewe rentekoers</i> is 13.07%</p>	$\checkmark \text{formula/for.}$ $\checkmark \text{substitution//verv.}$ $\checkmark \text{simplificationvereenv.}$ $\checkmark \text{answer/antw.} \quad (4)$
1.6	$A = P(1+i)^n$ $R\,221\,292,32 = R145\,000 \left(1 + \frac{r}{100}\right)^6$ $\sqrt[6]{\frac{R\,221\,292,32}{145\,000}} = 1 + \frac{r}{100}$ $\frac{r}{100} = 0,07300000324$ $r = 7,3\%$	$\checkmark \text{correct substitution into correct formula}$ $\checkmark n = 6$ \checkmark $\checkmark \sqrt[6]{\frac{R\,221\,292,32}{145\,000}} = 1 + \frac{r}{100}$ $\checkmark \text{answer/antw.} \quad (4)$

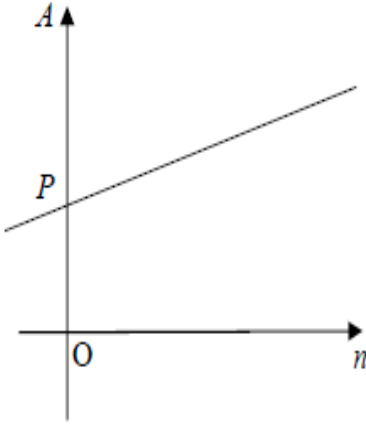
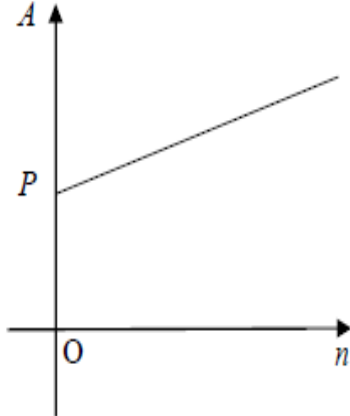
12.2 Class Activity/Home Activity B

1.1	$A = P(1-i)^n$ $20000 = 80000(1-i)^5$ $0,25 = (1-i)^5$ $\sqrt[5]{0,25} = 1-i$ $i = 1 - \sqrt[5]{0,25}$ $i = 0,24214417$ $i = 24,21\%$	<p>✓ substitution into correct formula/ verv.in korrekte vorm</p> <p>✓ simplification/vereenv</p> <p>✓ answer/antw.</p> <p>(3)</p>
1.2	$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$ $1 + i_{eff} = \left(1 + \frac{0,05}{4}\right)^4$ $i_{eff} = 0,050945336...$ <p>Effective rate = 5,09 % p.a.</p>	<p>✓ vorm/vorm</p> <p>✓ subst/verv</p> <p>✓ answer/antw.</p> <p>(3)</p>
1.3	$1 + i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m$ $1 + i_{eff} = \left(1 + \frac{0,115}{12}\right)^{12}$ $i_{eff} = \left(1 + \frac{0,115}{12}\right)^{12} - 1$ $i_{eff} = 12,13\%$	<p>✓ formula/form.</p> <p>✓ $i = \frac{0,115}{12}$</p> <p>✓ answer/antw.</p> <p>(3)</p>
1.4	$A = P(1-i)^n$ $= 4\,700(1-0,18)^4$ $= R\,2124,97$	<p>✓ formula/form.</p> <p>✓ substitution/verv.</p> <p>✓ answer/antw.</p> <p>(3)</p>

12.3 Class Activity/Home Activity C

3.1.	Quarterly interest rate/ <i>Kwartaallikse rentekoers</i> $= \frac{10\%}{4}$ $= 2,5\%$	✓ answer (1)
3.2.	$A = P(1+i)^n$ $= 5000\left(1 + \frac{2,5}{100}\right)^{2 \times 4}$ $= R6\,092,01$	✓ $n = 8$ ✓ $5000\left(1 + \frac{2,5}{100}\right)^{2 \times 4}$ ✓ answer (3)
3.3	$A = P(1+i)^n$ $23000 = 1570(1.12)^n$ $(1.12)^n = 14,64968153..$ $n \log(1,12) = \log 14,64968153..$ $n = 23,69 \text{ years} \quad (23,68701...)$ or $n = 24 \text{ years}$ or $n = 23 \text{ years } 8 \text{ months}$ or $n = 23,7 \text{ years}$ <div style="text-align: center;">OR</div> $A = P(1+i)^n$ $23000 = 1570\left(1 + \frac{12}{100}\right)^n$ $(1.12)^n = 14,64968153..$ $n \log(1,12) = \log 14,64968153..$ $n = 23,69 \text{ years} \quad (23,68701...)$ or $n = 24 \text{ years}$ or $n = 23 \text{ years } 8 \text{ months}$ or $n = 23,7 \text{ years}$	✓ formula ✓ substitution ✓ apply log function ✓ answer (4) ✓ formula ✓ substitution of $\frac{12}{100}$ ✓ apply log function ✓ answer (4)

12.4 Class Activity/Home Activity D

1.1.1	$A = P + (Pi)n$ which is a linear function of n .	✓ linear (1)
1.1.	 <p>Accept also:</p> 	✓ $P > 0$ ✓ slope > 0 (2)
1.1.	<p>The slope is Pi. Therefore this is the increase for A for each increase of 1 in n.</p> <p>OR</p> $ \begin{aligned} A(n+1) - A(n) &= [P + Pi(n+1)] - [P + Pin] \\ &= Pi[(n+1) - n] \\ &= Pi \end{aligned} $ <p>OR</p> $A(1) - A(0) = (P + Pi) - (P + 0) = Pi$	✓ Pi (1)

1.2	$A = P(1+i)^n$ $2P = P\left(1 + \frac{r}{4}\right)^{6 \times 4}$ $2 = \left(1 + \frac{r}{4}\right)^{24}$ $1 + \frac{r}{4} = 2^{\frac{1}{24}}$ $r = 4\left(2^{\frac{1}{24}} - 1\right)$ $r = 4\left(2^{\frac{1}{24}}\right) - 4$ $r = 0,1172 \dots$ <p>rate = 11,72% p.a. compounded quarterly</p> <p>OR</p> $A = P(1+i)^n$ $2P = P\left(1 + \frac{r}{400}\right)^{6 \times 4}$ $2 = \left(1 + \frac{r}{400}\right)^{24}$ $1 + \frac{r}{400} = 2^{\frac{1}{24}}$ $r = 400\left(2^{\frac{1}{24}} - 1\right)$ $r = 400\left(2^{\frac{1}{24}}\right) - 400$ $r = 11,72\% \text{ p.a.}$ <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p>Note: Penalty 1 for incorrect rounding off.</p> </div>	<p>✓ 2P</p> <p>✓ $\frac{r}{4}$ and 24</p> <p>✓ $1 + \frac{r}{4} = 2^{\frac{1}{24}}$</p> <p>✓ $r = 4\left(2^{\frac{1}{24}}\right) - 4$</p> <p>✓ answer (5)</p> <p>✓ 2P</p> <p>✓ $\frac{r}{400}$ and 24</p> <p>✓ $1 + \frac{r}{400} = 2^{\frac{1}{24}}$</p> <p>✓ $r = 400\left(2^{\frac{1}{24}}\right) - 400$</p> <p>✓ answer (5)</p>
1.3	$A = P(1+i)^n$ $1711,41 = 1430,77\left(1 + \frac{i}{12}\right)^{18}$ $\left(1 + \frac{i}{12}\right)^{18} = 1,196146131\dots$ $1 + \frac{i}{12} = 1,009999937\dots$ $i = 0,1199992431\dots$ <p>Rate = 12, 00% p.a. compounded monthly.</p> <div style="display: inline-block; vertical-align: top; margin-left: 20px;"> <p>OR</p> $\left[\frac{1711,41}{1430,77}\right]^{\frac{1}{18}} = 1,00999\dots$ $\therefore i = 12(1,01 - 1)$ $= 0,12$ $= 12\%$ </div>	<p>✓ substitution</p> <p>✓ $\left(1 + \frac{i}{12}\right)^{18} = 1,196146131\dots$</p> <p>✓ $1 + \frac{i}{12} = 1,009999937\dots$</p> <p>✓ answer (4)</p>

13. COGNITIVE LEVEL 1&2 SOLUTIONS (ANNUITIES)

13.1 Class Activity/Home Activity A

1.1	$A = P(1+i)^n$ $= 800000(1.08)^5$ $= R1175462,46$ $\therefore R1175462,46 - R200\ 000$ $= R975462,46$ <p>Some calculators give R 975 462,50</p>	<p>✓ substitution</p> <p>✓ R 1 175 462,46</p> <p>✓ R 975 462,46</p> <p>(3)</p> <p>Incorrect Formula: 0/3</p>
1.2	$P_v = \frac{x[1 - (1+i)^{-n}]}{i}$ $800000 = \frac{10000 \left[1 - \left(1 + \frac{0,14}{12} \right)^{-n} \right]}{\frac{0,14}{12}}$ $1 - \left(1 + \frac{0,14}{12} \right)^{-n} = \frac{14}{15} \quad (= 0,933333)$ $\left(1 + \frac{0,14}{12} \right)^{-n} = \frac{1}{15} \quad (= 0,06666666)$ $\log \left(1 + \frac{0,14}{12} \right)^{-n} = \log \frac{1}{15}$ $-n \log \left(1 + \frac{0,14}{12} \right) = \log \frac{1}{15} \quad \left(\begin{array}{l} -n = \frac{\log \frac{1}{15}}{\log \left(1 + \frac{0,14}{12} \right)} \\ = -233,47 \end{array} \right)$ $n = 233,47$ <p>\therefore the loan will be paid off at the end of the 234th month</p> <p>OR</p> <p>Balance outstanding after 233rd month</p> $= 800000 \left(1 + \frac{0,14}{12} \right)^{233} - \frac{10000 \left[\left(1 + \frac{0,14}{12} \right)^{233} - 1 \right]}{\frac{0,14}{12}}$ $= R4\ 660,04 \text{ which is less than R10\ 000}$ <p>Therefore the loan will be paid off after 234 months.</p> <p>OR</p> <p>Total value of the loan after 234 payments</p> $= \frac{10000 \left(1 - \left(1 + \frac{0,14}{12} \right)^{-234} \right)}{\frac{0,14}{12}}$ $= R800\ 350,21$ <p>> R800 000 and the differences is less than R10 000</p> <p>Therefore the loan will be paid off after 234 months.</p>	<p>✓ substitute into P_v</p> <p>✓ $i = \frac{0,14}{12}$</p> <p>✓ using logs</p> <p>✓ answer</p> <p>(4)</p> <p>✓ substitution into P formula</p> <p>✓ 234</p> <p>✓ answer</p> <p>✓ argument</p> <p>(4)</p> <p>✓ substitution into F formula</p> <p>✓ 234</p> <p>✓ answer</p> <p>✓ argument</p> <p>(4)</p>

1.3	$A = P(1-i)^n$ $\frac{P}{2} = P(1-0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log \frac{1}{2} = n \log 0,93$ $n = \frac{\log \frac{1}{2}}{\log 0,93}$ $= 9,55 \text{ years}$ <p style="text-align: center;">OR</p> $A = P(1-i)^n$ $\frac{P}{2} = P(1-0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log_{0,93} \frac{1}{2} = n$ $n = 9,55 \text{ years}$	$\checkmark A = \frac{P}{2}$ \checkmark subs into correct formula \checkmark log \checkmark answer
	<p>Note: If candidate interchanges A and P i.e. uses $P = \frac{A}{2}$: max 2/4 marks</p> <p>Note: If candidate uses incorrect formula: max 1/4 marks for $A = \frac{P}{2}$</p>	(4)

13.2 Class Activity/Home Activity B

1.1	$\text{Selling price / Verkoopprijs} = \frac{102\,000}{0,12}$ $= 850\,000$	$\checkmark 850\,000$ (1)
1.2	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $748\,000 = \frac{x \left[1 - \left(1 + \frac{0,09}{12} \right)^{-240} \right]}{\frac{0,09}{12}}$ $x = 6\,729,95$ <p style="text-align: center;">OR</p> $F_v = \frac{x[(1+i)^n - 1]}{i}$ $748\,000 \left(1 + \frac{0,09}{12} \right)^{240} = \frac{x \left[\left(1 + \frac{0,09}{12} \right)^{240} - 1 \right]}{\frac{0,09}{12}}$ $x = 6\,729,95$	$\checkmark P_v = 748\,000$ $\checkmark i = \frac{0,09}{12}$ $\checkmark n = -240$ $\checkmark x = R6\,729,95$ $\checkmark 748\,000 \left(1 + \frac{0,09}{12} \right)^{240}$ $\checkmark i = \frac{0,09}{12}$ $\checkmark n = 240$ $\checkmark x = R6\,729,95$ (4)
1.3	$\text{Balance} = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{6729,95 \left[1 - \left(1 + \frac{0,09}{12} \right)^{-155} \right]}{\frac{0,09}{12}}$ $x = 615\,509,74$	$\checkmark 6729,95$ $\checkmark n = -155$ $\checkmark R615\,509,74$ (3)

13.3 Class Activity/Home Activity C

3.1	$A = P(1 - i)^n$ $72\,500 = 145\,000(1 - i)^5$ $i = 1 - \sqrt[5]{\frac{72\,500}{145\,000}}$ $= 0,1294\dots$ <p>\therefore Rate of interest/<i>Rentekoers</i> is 12,94 % p.a./p.j.</p> <p>OR/OF</p> $(1 - i)^5 = \frac{1}{2}$ $\therefore i = 1 - \left(\frac{1}{2}\right)^{\frac{1}{5}}$ $i = 0,1294$ <p>\therefore Rate of interest/<i>Rentekoers</i> is 12,94 % p.a./p.j.</p>	<p>✓ substitution/<i>substitusie</i></p> <p>✓ writing in terms of <i>i</i> <i>herskryf in terme van i</i></p> <p>✓ answer/<i>antwoord</i> (3)</p> <p>✓ substitution/<i>substitusie</i></p> <p>✓ writing i.t.o <i>i</i></p> <p>✓ answer (3)</p>
3.2.	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $500\,000 = \frac{x\left[1 - \left(1 + \frac{0,12}{12}\right)^{-240}\right]}{\frac{0,12}{12}}$ $x = \frac{500\,000 \times \frac{0,12}{12}}{\left[1 - \left(1 + \frac{0,12}{12}\right)^{-240}\right]}$ $x = R5505,43$	<p>✓ $i = \frac{0,12}{12}$</p> <p>✓ $n = 240$</p> <p>✓ substitution into correct formula</p> <p>✓ answer/<i>antwoord</i> (4)</p>

3.2.	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $500000 = \frac{6000 \left[1 - \left(1 + \frac{0,12}{12} \right)^{-n} \right]}{\frac{0,12}{12}}$ $\frac{500000}{6000} \times 0,01 = 1 - (1,01)^{-n}$ $(1,01)^{-n} = 1 - \frac{5}{6}$ $-n = \frac{\log \frac{1}{6}}{\log 1,01}$ $n = 180,07$ <p>∴ Mphekgoane settles the loan in 181 months</p>	<p>✓ 6000</p> <p>✓ substitute into correct formula/substitusie in korrekte formule</p> <p>✓ use of logs/gebruik van logs</p> <p>✓ answer/antwoord (4)</p>
3.2.	<p>Malodi</p> <p>He is paying off his loan over a longer period thus more interest will be paid./Hy betaal sy lening oor 'n langer tydperk af, dus sal hy meer rente betaal.</p> <p>OR/OF</p> <p>Malodi</p> <p>He will pay/Hy betaal R5505,43 × 240 – R500 000 = R821 303,20</p> <p>She will pay between/Sy sal tussen R580 000 and/en R586 000,00 betaal.</p>	<p>✓ Malodi</p> <p>✓ reason/rede (2)</p> <p>✓ Malodi</p> <p>✓ reason/rede (2)</p>

13.4 Class Activity/Home Activity D

4.1.1	$.400 \times (44 \times 12)$ $= R211200$	$\checkmark R400 \times (44 \times 12)$ $\checkmark R211200$	(2)
4.2.	$F = \frac{x[(1+i)^n - 1]}{i}$ $400 \left[\frac{\left(1 + \frac{0,08}{12}\right)^{528} - 1}{\frac{0,08}{12}} \right]$ $= R1\,943\,524,42$	$\checkmark x = 400$ $\checkmark n = 528$ $\checkmark i = \frac{0,08}{12}$ $\checkmark \text{substitution into correct formula/substitusie in korrekte formule}$ $\checkmark \text{answer/antwoord}$	(5)
4.3	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $2000000 = \frac{x \left[1 - \left(1 + \frac{0,1}{12}\right)^{-300} \right]}{\frac{0,1}{12}}$ $x = R18\,174,01$ <p>OR/OF</p> $2000000 \left(1 + \frac{0,1}{12}\right)^{300} = \frac{x \left[\left(1 + \frac{0,1}{12}\right)^{300} - 1 \right]}{\frac{0,1}{12}}$ $x = R18174,01$	$\checkmark P = 2000000$ $\checkmark n = 300 \text{ and/en } i = \frac{0,1}{12}$ $\checkmark \text{substituting into correct formula/substitusie in korrekte formule}$ $\checkmark \text{answer/antwoord}$	(4)
		$\checkmark P = 2000000$ $\checkmark n = 300 \text{ and/en } i = \frac{0,1}{12}$ $\checkmark \text{equating/stel gelyk}$ $\checkmark \text{answer/antwoord}$	(4)

13.5 Class Activity/Home Activity E

1.1	$A = P(1+i)^n$ $12\ 146,72 = 10\ 000\left(1 + \frac{r}{12}\right)^{36}$ $\left(1 + \frac{r}{12}\right)^{36} = 1,214672$ $1 + \frac{r}{12} = \sqrt[36]{1,214672}$ $= 1,005416$ $\frac{r}{12} = 0,005416$ $r = 0,06500$ $r = 6,5\%$	$\checkmark \frac{r}{12}$ $\checkmark n = 36$ \checkmark correct substitution into formula $\checkmark 1 + \frac{r}{12} = \sqrt[36]{1,214672}$ $\checkmark 6,5\%$
1.2.1	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $235\ 000 = \frac{x\left[1 - \left(1 + \frac{0,11}{12}\right)^{-54}\right]}{\frac{0,11}{12}}$ $x = \frac{235\ 000 \times \frac{0,11}{12}}{\left[1 - \left(1 + \frac{0,11}{12}\right)^{-54}\right]}$ $= R5\ 536,95$ <p>His monthly instalment is R 5 536,95</p>	$\checkmark i = \frac{0,11}{12}$ $\checkmark n = 54$ \checkmark correct substitution in P \checkmark answer
1.2.2	<p>Amount paid for the year : $(5\ 536,95 \times 12) = R66\ 443,40$</p> $\text{Balance} = 235\ 000\left(1 + \frac{0,11}{12}\right)^{12} - \frac{5\ 536,95\left[\left(1 + \frac{0,11}{12}\right)^{12} - 1\right]}{\frac{0,11}{12}}$ $= 192\ 296,17$ <p>Interest = $(5\ 536,95 \times 12) - (235\ 000 - 192\ 296,17)$</p> $= 66\ 443,40 - 42\ 703,83$ $= 23\ 739,57$	$\checkmark R66\ 443,40$ $\checkmark 235\ 000\left(1 + \frac{0,11}{12}\right)^{12}$ $\checkmark \frac{5\ 536,95\left[\left(1 + \frac{0,11}{12}\right)^{12} - 1\right]}{\frac{0,11}{12}}$ $\checkmark R192\ 296,17$ $\checkmark R42\ 703,83$ $\checkmark R23\ 739,57$

15. PRESENT VALUE SOLUTIONS

15.1 Class Activity/Home Activity A

1.1	$0,10 \times R980\ 000$ $= R98\ 000$ $\therefore \text{Loan} = 980\ 000 - 98\ 000$ $= R882\ 000$	$\checkmark 0,10 \times R980\ 000$ $\checkmark R882\ 000 \quad (2)$
1.2	$P_v = \frac{x[1 - (1 + i)^{-n}]}{i}$ $882\ 000 = \frac{10\ 000 \left[1 - \left(1 + \frac{0,11}{12} \right)^{-n} \right]}{\frac{0,11}{12}}$ $\frac{1617}{2000} = 1 - \left(1 + \frac{0,11}{12} \right)^{-n}$ $\left(1 + \frac{0,11}{12} \right)^{-n} = \frac{383}{2000}$ $-n = \log_{\left(1 + \frac{0,11}{12} \right)} \frac{383}{2000}$ $-n = -181,14$ $n = 181,14$ $\therefore \text{It takes 182 months}$ <p style="text-align: center;">OR</p>	$\checkmark i = \frac{0,11}{12}$ \checkmark substitution into the correct formula $\checkmark \left(1 + \frac{0,11}{12} \right)^{-n} = \frac{383}{2000}$ \checkmark introducing logs $-n = \log_{\left(1 + \frac{0,11}{12} \right)} \frac{383}{2\ 000}$ $\text{or } -n = \log_{\left(\frac{1211}{1200} \right)} 0,1915$ $\text{or } -n = \frac{\log 0,1915}{\log 1,00916667}$ $\checkmark n = 181,14$ $\checkmark 182 \text{ months}$ <p style="text-align: right;">(6)</p>
	$P_v = \frac{x[1 - (1 + i)^{-n}]}{i}$ $882\ 000 = \frac{10\ 000 \left[1 - \left(1 + \frac{0,11}{12} \right)^{-n} \right]}{\frac{0,11}{12}}$ $\frac{383}{2000} = \left(1 + \frac{0,11}{12} \right)^{-n}$ $\log \left(\frac{383}{2000} \right) = \log \left(1 + \frac{0,11}{12} \right)^{-n}$ $\log \left(\frac{383}{2000} \right) = -n \log \left(1 + \frac{0,11}{12} \right)$ $-n = -181,14$ $n = 181,14$ $\therefore \text{It takes 182 months}$	$\checkmark i = \frac{0,11}{12}$ \checkmark substitution into the correct formula $\checkmark \frac{383}{2000} = \left(1 + \frac{0,11}{12} \right)^{-n}$ \checkmark introducing logs $-n = \frac{\log \left(\frac{383}{2000} \right)}{\log \left(1 + \frac{0,11}{12} \right)}$ $\text{or } -n = \frac{\log 0,1915}{\log 1,00916667}$ $\checkmark n = 181,14$ $\checkmark 182 \text{ months}$ <p style="text-align: right;">(6)</p>

1.3	$n = 181,1379918 - 90$ $= 91,1379918$ $P_v = \frac{x[1 - (1 + i)^{-n}]}{i}$ $P_v = \frac{10\,000 \left[1 - \left(1 + \frac{0,11}{12} \right)^{-91,1379918} \right]}{\frac{0,11}{12}}$ $= R615\,991,70$	<p>✓ n = 91,1379918</p> <p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p>(3)</p>

	<p>OR</p> $A = P(1 + i)^n$ $= 882\,000 \left(1 + \frac{0,11}{12}\right)^{90}$ $A = R2\,005\,069,01$ $F_v = \frac{x[(1 + i)^n - 1]}{i}$ $= \frac{10\,000 \left[\left(1 + \frac{0,11}{12}\right)^{90} - 1\right]}{\frac{0,11}{12}}$ $F_v = R1\,389\,077,31$ <p>Outstanding balance after 90 instalments:</p> $= R2\,005\,069,01 - R1\,389\,077,31$ $= R615\,991,70$	<p>✓ substitution into the correct formula (or 2 005 069,01)</p> <p>✓ substitution into the correct formula (or 1 389 077,31)</p> <p>✓ answer (R615 991,70)</p> <p>(3)</p>
1.4	$A = P(1 + i)^n$ $= 615\,991,70 \left(1 + \frac{0,11}{12}\right)^5$ $= 644\,747,02$ $P_v = \frac{x[1 - (1 + i)^{-n}]}{i}$ $644\,747,02 = \frac{x \left[1 - \left(1 + \frac{0,11}{12}\right)^{-87}\right]}{\frac{0,11}{12}}$ $x = R10\,786,84$	<p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p>✓ substitution into the correct formula</p> <p>✓ $n = 182 - 95 = 87$</p> <p>✓ answer</p> <p>(5) [16]</p>

15.2 Class Activity/Home Activity B

2.1	$A = P(1+i)^n$ $= 250000 \left(1 + \frac{0,15}{12}\right)^2$ $= R256\,289,06$	<p>✓ substituting i and n values in correct formula</p> <p>✓ answer</p> <p>(2)</p>
2.2	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $256\,289,06 = \frac{x \left[1 - \left(1 + \frac{0,15}{12}\right)^{-46}\right]}{\frac{0,15}{12}}$ $3203,6133 = x \left[1 - \left(1 + \frac{0,15}{12}\right)^{-46}\right]$ $x = R\,7\,359,79 \text{ per month}$ <p>OR/OF</p> $250\,000 = \frac{x \left(1 + \frac{0,15}{12}\right)^{-2} \left[1 - \left(1 + \frac{0,15}{12}\right)^{-46}\right]}{\frac{0,15}{12}}$ $x = R\,7\,359,79$	<p>✓ $i = \frac{0,15}{12}$</p> <p>✓ $n = 46$</p> <p>✓ substitution into correct formula</p> <p>✓ answer</p> <p>(4)</p> <p>✓ $i = \frac{0,15}{12}$</p> <p>✓ $n = 46$</p> <p>✓ substitution into correct formula</p> <p>✓ answer</p> <p>(4)</p>
2.3	$256\,289,06 = \frac{9\,000 \left[1 - \left(1 + \frac{0,15}{12}\right)^{-n}\right]}{\frac{0,15}{12}}$ $\left(1 + \frac{0,15}{12}\right)^{-n} = 0,6440429722$ $-n \log \left(1 + \frac{0,15}{12}\right) = \log 0,6440429722$ $n = 35,41872568 \text{ months/maande}$ <p>∴ 36 payments are required</p> <p>∴ 36 paaiente moet betaal word</p> <p>∴ Thabiso will pay his loan off 10 months sooner./Thabiso los sy lening 10 maande vroeër af.</p> <p>OR/OF</p>	<p>✓ $x = 9\,000$</p> <p>✓ substitute into correct formula</p> <p>✓ use of logs</p> <p>✓ $n = 35,42$</p> <p>✓ 10 months</p> <p>(5)</p>

	$256289,06 \left(1 + \frac{0,15}{12}\right)^n = \frac{9000 \left[\left(1 + \frac{0,15}{12}\right)^n - 1 \right]}{\frac{0,15}{12}}$ $3203,61325 \left(1 + \frac{0,15}{12}\right)^n = 9000 \left(1 + \frac{0,15}{12}\right)^n - 9000$ $9000 = 5796,38675 \left(1 + \frac{0,15}{12}\right)^n$ $n = \log_{\left(1 + \frac{0,15}{12}\right)} 1,5523691425$ $n = 35,41872568$ <p>\therefore 36 payments are required</p> <p>\therefore 36 <i>paaiente moet betaal word</i></p> <p>\therefore Thabiso will pay his loan off 10 months sooner./Thabiso <i>los sy lening 10 maande vroeër af.</i></p>	<p>✓ 9 000</p> <p>✓ substitute into correct formula</p> <p>✓ use of logs</p> <p>✓ $n = 35,42$</p> <p>✓ 10 months</p> <p>(5)</p>
2.4	<p>The balance of his loan after the 35th payment was made: <i>Die balans van sy lening nadat die 35^{ste} paaient betaal is:</i></p> $\text{Balance} = 256289,06 \left(1 + \frac{0,15}{12}\right)^{35} - \frac{9000 \left[\left(1 + \frac{0,15}{12}\right)^{35} - 1 \right]}{\frac{0,15}{12}}$ $= R \ 3 \ 735,45$ <p>Final instalment = $3 \ 735,45 \left(1 + \frac{0,15}{12}\right)$</p> $= R \ 3 \ 782,14$ <p>OR/OF</p> $P = \frac{x[1 - (1+i)^{-n}]}{i}$ <p>Final instalment</p> $= \frac{9 \ 000 \left[1 - \left(1 + \frac{0,15}{12}\right)^{-0,41872568} \right]}{\frac{0,15}{12}} \left(1 + \frac{0,15}{12}\right)$ $= R \ 3 \ 782,14$ <p>OR/OF</p>	<p>✓ $256289,06 \left(1 + \frac{0,15}{12}\right)^{35}$</p> <p>✓ $\frac{9000 \left[\left(1 + \frac{0,15}{12}\right)^{35} - 1 \right]}{\frac{0,15}{12}}$</p> <p>✓ $3 \ 735,45 \left(1 + \frac{0,15}{12}\right)$</p> <p>✓ answer</p> <p>(4)</p> <p>✓ 0,41872568</p> <p>✓ $\frac{9 \ 000 \left[1 - \left(1 + \frac{0,15}{12}\right)^{-0,41872568} \right]}{\frac{0,15}{12}}$</p> <p>✓ $\times \left(1 + \frac{0,15}{12}\right)$</p> <p>✓ answer</p> <p>(4)</p>
	$\text{Balance} = 256289,06 \left(1 + \frac{0,15}{12}\right)^{36} - \frac{9000 \left[\left(1 + \frac{0,15}{12}\right)^{36} - 1 \right]}{\frac{0,15}{12}}$ $= R \ - \ 5 \ 217,86$ <p>Final payment = $9 \ 000 - 5217,86$</p> $= R \ 3 \ 782,14$	<p>✓ $256289,06 \left(1 + \frac{0,15}{12}\right)^{36}$</p> <p>✓ $\frac{9000 \left[\left(1 + \frac{0,15}{12}\right)^{36} - 1 \right]}{\frac{0,15}{12}}$</p> <p>✓ $9 \ 000 - 5217,86$</p> <p>✓ answer</p> <p>(4)</p> <p>[15]</p>

15.3 Class Activity/Home Activity C

3.1.1	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $235\,000 = \frac{x \left[1 - \left(1 + \frac{0,11}{12} \right)^{-54} \right]}{\frac{0,11}{12}}$ $x = \frac{235\,000 \times \frac{0,11}{12}}{\left[1 - \left(1 + \frac{0,11}{12} \right)^{-54} \right]}$ $= R5\,536,95$ <p>His monthly instalment is R 5 536,95</p>	<p>✓ $i = \frac{0,11}{12}$ ✓ $n = 54$ ✓ correct substitution in P</p> <p>✓ answer</p> <p>(4)</p>
3.1.2	<p>Amount paid for the year : $(5\,536,95 \times 12) = R66\,443,40$</p> $\text{Balance} = 235\,000 \left(1 + \frac{0,11}{12} \right)^{12} - \frac{5\,536,95 \left[\left(1 + \frac{0,11}{12} \right)^{12} - 1 \right]}{\frac{0,11}{12}}$ $= 192\,296,17$ <p>Interest = $(5\,536,95 \times 12) - (235\,000 - 192\,296,17)$</p> $= 66\,443,40 - 42\,703,83$ $= 23\,739,57$ <p>OR/OF</p>	<p>✓ R66 443,40</p> <p>✓ $235\,000 \left(1 + \frac{0,11}{12} \right)^{12}$</p> <p>✓ $\frac{5\,536,95 \left[\left(1 + \frac{0,11}{12} \right)^{12} - 1 \right]}{\frac{0,11}{12}}$</p> <p>✓ R192 296,17</p> <p>✓ R42 703,83 ✓ R23 739,57</p> <p>OR/OF</p>

<p>Total amount paid in first year = $R\ 5\ 536,95 \times 12$ $= R66\ 443,40$</p> <p>Balance on loan after 1 year = P of remaining installments</p> $P = \frac{x[1 - (1+i)^{-n}]}{i}$ $= \frac{5\ 536,95 \left[1 - \left(1 + \frac{0,11}{12} \right)^{-42} \right]}{\frac{0,11}{12}}$ $= R192\ 296,20$ <p>Amount paid off in the first year: $R235\ 000 - R192\ 296,20 = R42\ 703,80$</p> <p>Amount of interest = $R66\ 443,40 - R42\ 703,80$ $= R23\ 739,60$</p> <p>OR/OF</p> $P = \frac{5536,95 \left[1 - \left(1 + \frac{0,11}{12} \right)^{-12} \right]}{\frac{0,11}{12}}$ $= R\ 62\ 648,18$ <p>$235\ 000 - 62\ 648,18 = R172\ 351,82$</p> <p>After 12 months, money owed on house is</p> $172\ 351,82 \left(1 + \frac{0,11}{12} \right)^{12}$ $= 192\ 296,17$ <p>Amount paid after 12 months is</p> $5\ 536,95 \times 12 = R\ 66\ 443,40$ <p>Amount of interest paid:</p> $R\ 66\ 443,40 - (235\ 000 - 192\ 296,17)$ $= R\ 23\ 739,57$	<p>✓ R66 443,40</p> <p>✓ $n = -42$</p> <p>✓ substitution into correct formula</p> <p>✓ R192 296,20</p> <p>✓ R42 703,80</p> <p>✓ R23 739,60</p> <p>OR/OF (6)</p> <p>✓ R62 648,18</p> <p>✓ R172 351,82</p> <p>✓ R192 296,17</p> <p>✓ R66 443,40</p> <p>✓ $235\ 000 - 192\ 296,17$</p> <p>✓ R23 739,57</p> <p>(6)</p> <p>[15]</p>
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15.4 Class Activity/Home Activity D

4.1.1	<p>After eleven months, Genevieve will owe/ <i>Na elf maande skuld Genevieve</i></p> $A = 82\,000 \left(1 + \frac{0,15}{12}\right)^{11}$ $= R\,94\,006,79$	<p>✓ $n = 11$ ✓ correct substitution into correct formula ✓ answer</p> <p>(3)</p>
4.1.2	$P = \frac{x \left[1 - (1+i)^{-n}\right]}{i}$ $94\,006,79 = \frac{3\,200 \left[1 - \left(1 + \frac{0,15}{12}\right)^{-n}\right]}{\frac{0,15}{12}}$ $\frac{94\,006,79}{3\,200} \times \frac{0,15}{12} = 1 - \left(1 + \frac{0,15}{12}\right)^{-n}$ $\left(1 + \frac{0,15}{12}\right)^{-n} = 1 - 0,3672147...$ $-n \log \left(1 + \frac{0,15}{12}\right) = \log 0,6327852...$ $-n = -36,8382...$ $n = 36,84$ <p>Genevieve will have to pay 36 installments of R3 200</p>	<p>✓ 94006,79 ✓ substitute into correct formula</p> <p>✓ correct use of logs (logs to be defined)</p> <p>✓ $n = 36,84$ ✓ 36 installments</p> <p>(5)</p>

4.1.3	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $= \frac{3200 \left[1 - \left(1 + \frac{0,15}{12} \right)^{-0,83826912} \right]}{\frac{0,15}{12}}$ $P = 2652$ <p>Outstanding balance after 36 installments is R2 652 Final payment will be:</p> $A = 2652,00 \left(1 + \frac{0,15}{12} \right)^1$ $= \text{R } 2685,00$ <p>OR/OF</p> $\text{Balance : } 94006,79 \left(1 + \frac{0,15}{12} \right)^{36} - \frac{3200 \left[\left(1 + \frac{0,15}{12} \right)^{36} - 1 \right]}{\frac{0,15}{12}}$ $= \text{R2 } 651,72$ <p>Final payment will be:</p> $A = 2651,72 \left(1 + \frac{0,15}{12} \right)^1$ $= \text{R } 2685,00$	$\checkmark n = -0,83826912$ \checkmark substitute into correct formula \checkmark answer $\checkmark 2652,00 \left(1 + \frac{0,15}{12} \right)^1$ \checkmark answer <p>OR/OF</p> $\checkmark 94006,79 \left(1 + \frac{0,15}{12} \right)^{36}$ $\checkmark \frac{3200 \left[\left(1 + \frac{0,15}{12} \right)^{36} - 1 \right]}{\frac{0,15}{12}}$ $\checkmark 2651,72$ $\checkmark 2651,72 \left(1 + \frac{0,15}{12} \right)^1$ \checkmark answer <div style="text-align: right;">(5) [16]</div>
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4.2.1	$P = \frac{x[1 - (1 + i)^{-n}]}{i}$ $525\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,1}{12} \right)^{-n} \right]}{\frac{0,1}{12}}$ $\frac{35}{48} = 1 - \left(1 + \frac{0,1}{12} \right)^{-n}$ $-n \log \left(1 + \frac{0,1}{12} \right) = \log \frac{13}{48}$ $-n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12} \right)}$ $n = 157,40$ $n = 158 \text{ payments}$ <p>OR/OF</p>	$\checkmark \frac{0,1}{12}$ \checkmark substitution into the correct formula \checkmark simplification \checkmark use of logs \checkmark answer (5) OR/OF
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	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $525\,000 = \frac{6\,000 \left[1 - \left(1 + \frac{0,1}{12} \right)^{-12n} \right]}{\frac{0,1}{12}}$ $\frac{35}{48} = 1 - \left(1 + \frac{0,1}{12} \right)^{-12n}$ $-12n \log \left(1 + \frac{0,1}{12} \right) = \log \frac{13}{48}$ $-12n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12} \right)}$ $n = \frac{\log \frac{13}{48}}{\log \left(1 + \frac{0,1}{12} \right)} \times \frac{1}{12}$ $n = 13,11686841$ <p>Number of payments = $13,11686841 \times 12 = 157,40$ $n = 158$ payments</p>	<p>✓ $\frac{0,1}{12}$</p> <p>✓ substitution into the correct formula</p> <p>✓ simplification</p> <p>✓ use of logs</p> <p>✓ answer</p> <p>(5)</p>
4.2.2	<p>Difference: $R6\,000 - R5\,066,36 = R933,64$</p> $F = \frac{x[(1+i)^n - 1]}{i}$ $F = \frac{933,64 \left[\left(1 + \frac{0,1}{12} \right)^{108} - 1 \right]}{\frac{0,1}{12}}$ <p>= R162 503,51</p>	<p>✓ R933,64</p> <p>✓ $n = 108$</p> <p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p>(4)</p>

16. FUTURE VALUE SOLUTIONS

16.1 Class Activity/Home Activity A

1.1.1	$R400 \times (44 \times 12)$ $= R211200$	$\checkmark R400 \times (44 \times 12)$ $\checkmark R211200$	(2)
1.1.2	$F = \frac{x[(1+i)^n - 1]}{i}$ $400 \left[\left(1 + \frac{0,08}{12} \right)^{528} - 1 \right]$ $= \frac{0,08}{12}$ $= R1\,943\,524,42$	$\checkmark x = 400$ $\checkmark n = 528$ $\checkmark i = \frac{0,08}{12}$ $\checkmark \text{substitution into correct formula/substitusie in korrekte formule}$ $\checkmark \text{answer/antwoord}$	(5)
1.1.3	$P = \frac{x[1 - (1+i)^{-n}]}{i}$ $2000000 = \frac{x \left[1 - \left(1 + \frac{0,1}{12} \right)^{-300} \right]}{\frac{0,1}{12}}$ $x = R18\,174,01$ <p>OR/OF</p> $2000000 \left(1 + \frac{0,1}{12} \right)^{300} = \frac{x \left(\left(1 + \frac{0,1}{12} \right)^{300} - 1 \right)}{\frac{0,1}{12}}$ $x = R18174,01$	$\checkmark P = 2000000$ $\checkmark n = 300 \text{ and/en } i = \frac{0,1}{12}$ $\checkmark \text{substituting into correct formula/substitusie in korrekte formule}$ $\checkmark \text{answer/antwoord}$	(4)
1.2	<p>Let P_X and P_Y be the populations of the two towns at the beginning of 2010./Laat P_X en P_Y die bevolkings wees van die twee dorpe aan die begin van 2010.</p> $A_X = A_Y$ $P_X(1 - 0,08)^3 = P_Y(1 + 0,12)^3$ $\frac{P_X}{P_Y} = \frac{(1 + 0,12)^3}{(1 - 0,08)^3}$ $= \frac{1,404...}{0,778...}$ $= 1,8:1$	$\checkmark \text{equating/stel gelyk}$ $\checkmark A_X = P_X(1 - 0,08)^3$ $\checkmark A_Y = P_Y(1 + 0,12)^3$ $\checkmark \text{answer/antwoord}$	(4) [15]

16.2 Class Activity/Home Activity B

2.1.1	$F = \frac{x[(1+i)^n - 1]}{i}$ $= \frac{1\,000 \left[\left(1 + \frac{0,075}{12} \right)^{145} - 1 \right]}{\frac{0,075}{12}}$ $= R234\,888,53$	<p>✓ $n = 145$</p> <p>✓ $i = \frac{0,075}{12}$</p> <p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p>(4)</p>
2.1.2	$A = P(1+i)^n$ $= 234\,888,53 \left(1 + \frac{0,075}{12} \right)^{12}$ $= R253\,123,54$	<p>✓ substitution into the correct formula</p> <p>✓ answer</p> <p>(2)</p>

16.3 Class Activity/Home Activity C

3.1	R450 000	✓ answer
3.2	$A = P(1-i)^n$ $f(x) = 450\,000(1-i)^x$ $243\,736,90 = 450\,000(1-i)^4$ $i = 1 - \sqrt[4]{\frac{243\,736,90}{450\,000}}$ $i = 0,1421$ <p>The rate of depreciation is 14,21% p.a. <i>Die waardeverminderingkoers is 14,21% p.j.</i></p>	<p>✓ substitution of 450 000 into correct formula</p> <p>✓ substitution of (4; 243 736,90) into correct formula</p> <p>✓ making i the subject</p> <p>✓ answer</p> <p>(4)</p>
3.3	<p>At T :</p> $A = P(1+i)^n$ $g(x) = 450\,000(1+i)^x$ $a = 450\,000(1+0,081)^4$ $= R614\,490,66$	<p>✓ $i = 0,081$ & $n = 4$</p> <p>✓ correct substitution into formula</p> <p>✓ answer</p> <p>(3)</p>

3.4	<p>Future Value = R614 490,66 – R243 736,90 = R370 753,76</p> <p>Let x be the value of monthly payment</p> $F_v = \frac{x[(1+i)^n - 1]}{i}$ $370753,76 = \frac{x \left[\left(1 + \frac{0,062}{12} \right)^{36} - 1 \right]}{\frac{0,062}{12}}$ $x = R9397,11$	<p>✓ R370 753,76</p> <p>✓ $i = \frac{0,062}{12}$</p> <p>✓ $n = 36$</p> <p>✓ substitution into correct formula</p> <p>✓ answer</p> <p>(5)</p> <p>[13]</p>
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16.4 Class Activity/Home Activity D

4.1.1	$A = 150\,000(1 - 0,2)^2$ $= R96\,000$	<p>✓ $n = 2$</p> <p>✓ 150 000 in correct formula</p> <p>✓ 96 000</p> <p>(3)</p>
4.1.2	$150\,000(1 - 0,2)^n = 49\,152$ $(0,8)^n = \frac{1024}{3125}$ $n \log(0,8) = \log \frac{1024}{3125}$ $n = 5$ <p>The machine will need to be replaced at the beginning of 2020 / <i>Masjien moet aan die begin van 2020 vervang word</i></p> <p>OR / OF</p> $150\,000(1 - 0,2)^n = 49\,152$ $(0,8)^n = \frac{1024}{3125}$ $n = \log_{0,8} \frac{1024}{3125}$ $n = 5$ <p>The machine will need to be replaced at the beginning of 2020 / <i>Masjien moet aan die begin van 2020 vervang word</i></p>	<p>✓ $150\,000(1 - 0,2)^n = 49\,152$</p> <p>✓ $n \log(0,8) = \log \frac{1024}{3125}$</p> <p>✓ $n = 5$</p> <p>✓ 2020</p> <p>(4)</p> <p>✓ $150\,000(1 - 0,2)^n = 49\,152$</p> <p>✓ $n = \log_{0,8} \frac{1024}{3125}$</p> <p>✓ $n = 5$</p> <p>✓ 2020</p> <p>(4)</p>
4.1.3	$R230\,000 - R49\,152$ $= R230\,848$ $230\,848 = \frac{x \left[\left(1 + \frac{0,085}{4} \right)^{20} - 1 \right]}{\frac{0,085}{4}}$ $x = R9\,383,26$	<p>✓ R230 848</p> <p>✓ $i = \frac{0,085}{4} = 0,02125$</p> <p>and $n = 20$</p> <p>✓ subs into correct formula</p> <p>✓ R 9 383,26</p> <p>(4)</p>

16.5 Class Activity/Home Activity E

5.1.1	$F = \frac{x[(1+i)^n - 1]}{i}$ $F = \frac{15\,000 \left[\left(1 + \frac{0,088}{4} \right)^{16} - 1 \right]}{\frac{0,088}{4}}$ $F = \text{R}283\,972,28$	<p>✓ $\frac{0,088}{4}$ and $n = 16$</p> <p>✓ substitution into correct formula</p> <p>✓ answer</p> <p>(3)</p>
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5.1.2	$A = \text{R}283\,972,28 - 100\,000 \left(1 + \frac{0,088}{4} \right)^4$ $= \text{R}174\,877,60$ <p>OR/OF Amount at end of 3 years:</p> $F = \frac{15\,000 \left[\left(1 + \frac{0,088}{4} \right)^{12} - 1 \right]}{\frac{0,088}{4}} - 100\,000$ $= \text{R}103\,459,12$ <p>Amount at end of 4 years:</p> $P(1+i)^n + \frac{x[(1+i)^n - 1]}{i}$ $= 103\,459,12 \left(1 + \frac{0,088}{4} \right)^4 + \frac{15\,000 \left[\left(1 + \frac{0,088}{4} \right)^4 - 1 \right]}{\frac{0,088}{4}}$ $= \text{R}174\,877,60$	<p>✓ future value – amount including interest</p> <p>✓ $100\,000 \left(1 + \frac{0,088}{4} \right)^4$</p> <p>✓ answer</p> <p>(3)</p> <p>OR/OF</p> <p>✓ R15 000 including interest – R100 000</p> <p>✓ $\left(1 + \frac{0,088}{4} \right)^4$ on P and x in F_v</p> <p>✓ method</p> <p>(3)</p>
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17. SIMPLE AND COMPOUND INTEREST SOLUTIONS

17.1 Class Activity/Home Activity A

1.1	<p>Kuda : $A = P(1 + in)$ $= 5\,000(1 + 0,083 \times 4)$ $= R6\,660,00$ Final Answer : $R6\,660,00 + R266,40$ $= R6\,926,40$</p> <p>OR/OF Kuda : $A = P(1 + in) \times 1,04$ $= 5\,000(1 + 0,083 \times 4) \times 1,04$ $= R6\,926,40$</p> <p>Thabo : $A = P(1 + i)^n$ $= 5\,000 \left(1 + \frac{0,081}{12}\right)^{12 \times 4}$ $= R6\,905,71$</p> <p>Kuda will have a better investment</p>	<p>✓ substitution into the correct formula</p> <p>✓ final answer</p> <p>OR/OF ✓ substitution into the correct formula ✓ final answer</p> <p>✓ substitution into the correct formula ✓ answer</p> <p>✓ conclusion (5)</p>
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17.2 Class Activity/Home Activity B

2.1	$A = P(1 - i)^n$ $\frac{P}{2} = P(1 - 0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log \frac{1}{2} = n \log 0,93$ $n = \frac{\log \frac{1}{2}}{\log 0,93}$ $= 9,55 \text{ years}$	<p>OR</p> $A = P(1 - i)^n$ $\frac{P}{2} = P(1 - 0,07)^n$ $\frac{1}{2} = 0,93^n$ $\log_{0,93} \frac{1}{2} = n$ $n = 9,55 \text{ years}$	<p>✓ $A = \frac{P}{2}$</p> <p>✓ subs into correct formula</p> <p>✓ log</p> <p>✓ answer (4)</p>
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Note:
If candidate interchanges A and P
i.e. uses $P = \frac{A}{2}$: max 2/4 marks

Note:
If candidate uses incorrect formula: max 1/4 marks
for $A = \frac{P}{2}$

<p>2.2</p>	<p>Radesh:</p> $A = P(1 + in)$ $= 6\,000(1 + 0,085 \times 5)$ $= 8\,550$ <p style="text-align: center;">OR</p> $A = 6\,000 + 8,5\% \text{ of } 6000 \times 5$ $= 6000 + 510 \times 5$ $= 6000 + 2550$ $= 8\,550$ <p>Bonus = $0,05 \times 6\,000$</p> $= 300$ <p>Received = $8\,550 + 300$</p> $= R8\,850$ <p>Thandi:</p> $A = P(1 + i)^n$ $= 6\,000 \left(1 + \frac{0,08}{4}\right)^{20}$ $= R8\,915,68$ <p>Thandi's investment is bigger.</p>	<p>✓ 8 550</p> <p>✓ R8 850</p> <p>✓ $n = 20$</p> <p>✓ $i = \frac{0,08}{4}$</p> <p>✓ answer</p> <p>✓ choice made</p> <p style="text-align: right;">(6)</p>
<p>2.3</p>	<p>F_v = initial deposit with interest + annuity</p> $= 1\,000 \left(1 + \frac{0,15}{12}\right)^{18} + 700 \left(\frac{\left(1 + \frac{0,15}{12}\right)^{18} - 1}{\frac{0,15}{12}}\right)$ $= 1\,250,58 + 14\,032,33$ $= R15\,282,91$ <p>OR</p> <p>F_v = initial deposit with interest + annuity</p> $= 1\,000 \left(1 + \frac{0,15}{12}\right)^{18} + 700 \left(\frac{1 - \left(1 + \frac{0,15}{12}\right)^{-18}}{\frac{0,15}{12}}\right) \left(1 + \frac{0,15}{12}\right)^{18}$ $= 1\,250,58 + 11\,220,68 \left(1 + \frac{0,15}{12}\right)^{18}$ $= 1\,250,58 + 14\,032,33$ $= R15\,282,91$	<p>✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or 0,0125</p> <p>✓ $n = 18$</p> <p>✓ $n = 18$</p> <p>✓ $1\,000 \left(1 + \frac{0,15}{12}\right)^{18}$</p> <p>✓ $700 \left(\frac{\left(1 + \frac{0,15}{12}\right)^{18} - 1}{\frac{0,15}{12}}\right)$</p> <p>✓ answer</p> <p style="text-align: right;">(6)</p> <p>✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or 0,0125</p> <p>✓ $n = 18$</p> <p>✓ $n = 18$</p> <p>✓ $1\,000 \left(1 + \frac{0,15}{12}\right)^{18}$</p> <p>✓ $700 \left(\frac{1 - \left(1 + \frac{0,15}{12}\right)^{-18}}{\frac{0,15}{12}}\right) \left(1 + \frac{0,15}{12}\right)^{18}$</p> <p>✓ answer</p> <p style="text-align: right;">(6)</p>

	<p>OR</p> $F_v = 300 \left(1 + \frac{0,15}{12} \right)^{18} + 700 \left(\frac{\left(1 + \frac{0,15}{12} \right)^{19} - 1}{\frac{0,15}{12}} \right)$ $= 375,17 + 14\,907,74$ $= \text{R}15\,282,91$	<p>✓ $i = \frac{0,15}{12}$ or $\frac{1}{80}$ or 0,0125</p> <p>✓ $n = 19$ (corresponding to 700)</p> <p>✓ $n = 18$ (corresponding to 300)</p> <p>✓ $300 \left(1 + \frac{0,15}{12} \right)^{18}$</p> <p>✓ $700 \left(\frac{\left(1 + \frac{0,15}{12} \right)^{19} - 1}{\frac{0,15}{12}} \right)$</p> <p>✓ answer</p> <p style="text-align: right;">(6) [16]</p>
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Capricorn North
District
Mathematics
Calculus
Grade 12
Manual marking Guide
Level 1 & 2

5. ANSWERS FOR FIRST PRINCIPLE

1	$f(x) = x^2 + x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h}$ $f'(x) = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h + 1)}{h}$ $\therefore f'(x) = 2x + 1$ <p>OR/OF</p> $f(x) = x^2 + x$ $f(x+h) = (x+h)^2 + (x+h) = x^2 + 2xh + h^2 + x + h$ $f(x+h) - f(x) = x^2 + 2xh + h^2 + x + h - x^2 - x$ $= 2xh + h^2 + h$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{2xh + h^2 + h}{h}$ $= \lim_{h \rightarrow 0} \frac{h(2x + h + 1)}{h}$ $\therefore f'(x) = 2x + 1$	<p>✓ substitution into the formula</p> <p>✓ $x^2 + 2xh + h^2 + x + h$</p> <p>✓ $2xh + h^2 + h$</p> <p>✓ common factor</p> <p>✓ answer (5)</p> <p>OR/OF</p> <p>✓ $x^2 + 2xh + h^2 + x + h$</p> <p>✓ $2xh + h^2 + h$</p> <p>✓ substitution into the formula</p> <p>✓ common factor</p> <p>✓ answer (5)</p>
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Penalty of – 1 for notation only in 7.1

2	$f(x) = 2x^2 - 1$ $f(x+h) = 2(x+h)^2 - 1$ $= 2(x^2 + 2xh + h^2) - 1$ $= 2x^2 + 4xh + 2h^2 - 1$ $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 - 1 - (2x^2 - 1)$ $= 2x^2 + 4xh + 2h^2 - 1 - 2x^2 + 1$ $= 4xh + 2h^2$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{4xh + 2h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(4x + 2h)}{h}$ $= \lim_{h \rightarrow 0} (4x + 2h)$ $= 4x$	<p>✓ $2x^2 + 4xh + 2h^2 - 1$</p> <p>✓ $4xh + 2h^2$</p> <p>✓ substitution</p> <p>✓ simplification</p> <p>✓ answer</p> <p>(5)</p>
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3	$f(x) = 4 - 7x$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{4 - 7(x+h) - (4 - 7x)}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-7)}{h}$ $= -7$	<p>✓ $4 - 7(x+h)$</p> <p>✓ substitution</p> <p>✓ simplification</p> <p>✓ answer</p> <p>(4)</p>
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4	$f(x) = -\frac{2}{x}$ $f(x+h) = -\frac{2}{(x+h)}$ $f(x+h) - f(x) = -\frac{2}{(x+h)} - \left(-\frac{2}{x}\right)$ $= \frac{-2x + 2(x+h)}{x(x+h)}$ $= \frac{-2x + 2x + 2h}{x(x+h)}$ $= \frac{2h}{x(x+h)}$ $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{2h}{x(x+h)}}{h}$ $= \lim_{h \rightarrow 0} \left(\frac{2}{x^2 + xh} \right)$ $= \frac{2}{x^2}$	<p>✓ substitution</p> <p>✓ simplification</p> <p>✓ formula</p> <p>✓ common factor</p> <p>✓ answer</p> <p>(5)</p>
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5	$f(x+h) = (x+h)^3 = (x^2 + 2xh + h^2)(x+h)$ $= x^3 + x^2h + 2x^2h + 2xh^2 + h^2x + h^3$ $= x^3 + 3x^2h + 3xh^2 + h^3$ $f(x+h) - f(x) = x^3 + 3x^2h + 3xh^2 + h^3 - x^3$ $= 3x^2h + 3xh^2 + h^3$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$ $= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$ $= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2)$ $= 3x^2$ <p>OR/OF</p> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)(x+h)^2 - x^3}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)(x^2 + 2xh + h^2) - x^3}{h}$ $= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$ $= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$ $= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2)$ $= 3x^2$	<p>✓ simplifying/vereenvouding</p> <p>✓ formula/formule</p> <p>✓ subst. into formula/subst. in formule</p> <p>✓ factorization/factoriseren</p> <p>✓ answer/antwoord</p> <p>(5)</p> <p>✓ formula/formule</p> <p>✓ subst. into formula/subst. in formule</p> <p>✓ simplifying/vereenvoudiging</p> <p>✓ factorization/factoriseren</p> <p>✓ answer/antwoord</p> <p>(5)</p>
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6. ANSWERS FOR USING RULES TO DEFERENTIATE.

$$1. \quad f(x) = 2x^5 - 3x^4 + 8x$$

$$f'(x) = 10x^4 - 12x^3 + 8 \checkmark \checkmark \checkmark \quad (3)$$

$$2. \quad a. \quad f(x) = \frac{x+2x^3}{\sqrt{x}}$$

$$b. \quad f(x) = \frac{x}{x^{\frac{1}{2}}} + \frac{2x^3}{x^{\frac{1}{2}}} \checkmark$$

$$f(x) = x^{\frac{1}{2}} + 2x^{\frac{5}{2}} \checkmark$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 5x^{\frac{3}{2}} \checkmark \checkmark \quad (4)$$

3. a

$\frac{d}{dx} \left(\sqrt[5]{x^2} + x^3 \right)$ $= \frac{d}{dx} \left(x^{\frac{2}{5}} + x^3 \right)$ $\frac{dy}{dx} = \frac{2}{5}x^{-\frac{3}{5}} + 3x^2$	$\checkmark x^{\frac{2}{5}}$ $\checkmark \frac{2}{5}x^{-\frac{3}{5}} \quad \checkmark 3x^2$
(3)	

b.

$f(x) = \frac{4x^2 - 9}{4x + 6}$ $= \frac{(2x-3)(2x+3)}{2(2x+3)}$ $= \frac{2x-3}{2}$ $= x - \frac{3}{2}$ $f'(x) = 1$	$\checkmark (2x-3)(2x+3)$ $\checkmark 2(2x+3)$ $\checkmark \text{ simplification to two separate terms}$ $\checkmark \text{ answer}$
(4)	

4. a.

$y = 3x^3 + 6x^2 + x - 4$ $\frac{dy}{dx} = 9x^2 + 12x + 1$	$\checkmark 9x^2$ $\checkmark 12x$ $\checkmark 1$
(3)	

b.

$y(x-1) = 2x(x-1)$ $y = \frac{2x(x-1)}{x-1} \text{ if } x \neq 1$ $y = 2x$ $\frac{dy}{dx} = 2$	$\checkmark y(x-1)$ $\checkmark 2x(x-1)$ $\checkmark y = 2x$ $\checkmark \text{answer}$
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(4)

c.

$y = 4x^8 + \sqrt{x^3}$ $= 4x^8 + x^{\frac{3}{2}}$ $\frac{dy}{dx} = 32x^7 + \frac{3}{2}x^{\frac{1}{2}}$	$\checkmark x^{\frac{3}{2}}$ $\checkmark 32x^7$ $\checkmark \frac{3}{2}x^{\frac{1}{2}}$
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(3)

d.

$y = \left(x^2 - \frac{1}{x^2}\right)^2$ $y = x^4 - 2 + \frac{1}{x^4}$ $= x^4 - 2 + x^{-4}$ $\frac{dy}{dx} = 4x^3 - 4x^{-5}$	$\checkmark x^4 - 2 + \frac{1}{x^4}$ $\checkmark 4x^3$ $\checkmark -4x^{-5}$
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(3)

e.

$y = \frac{1}{4}x^2 - 2x$ $\frac{dy}{dx} = \frac{1}{2}x - 2$	$\checkmark \frac{1}{2}x$ $\checkmark -2$
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(2)

f.

$y = 2x^{-4} - \frac{x}{5}$ $\frac{dy}{dx} = -8x^{-5} - \frac{1}{5}$	$\checkmark -8x^{-5}$ $\checkmark -\frac{1}{5}$	(2)
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g. $xy = 5$

$$y = \frac{5}{x} \checkmark$$

$$y = 5x^{-1} \checkmark$$

$$\frac{dy}{dx} = -\frac{5}{x^2} \checkmark$$

(3)

h. $y = \frac{1-2x+x^2}{x}$

$$y = \frac{1}{x} - \frac{2x}{x} + \frac{x^2}{x} \checkmark$$

$$y = x^{-1} - 2 + x \checkmark$$

$$\frac{dy}{dx} = -x^{-2} + 1 \checkmark \checkmark$$

(4)

5

$y = ax^2 + a$ $\frac{dy}{dx} = 2ax + 0$ $\frac{dy}{dx} = 2ax$	$\checkmark 2ax$	(1)
$y = ax^2 + a$ $\frac{dy}{da} = x^2 + 1$	$\checkmark \checkmark$ answer	(2)

6.

$\begin{aligned} &D_x \left[\frac{(x-1)(x^2+x+1)}{x-1} \right] \\ &= D_x [x^2 + x + 1] \\ &= 2x + 1 \end{aligned}$	<p>✓ factorisation</p> <p>✓ $x^2 + x + 1$</p> <p>✓ $2x + 1$</p>
OR/OF	
By using the quotient rule (with is not part of CAPS):	
$\begin{aligned} &D_x \left[\frac{x^3 - 1}{x - 1} \right] \\ &= \frac{3x^2(x-1) - (x^3 - 1)}{(x-1)^2} \end{aligned}$	<p>✓✓✓</p> $\frac{3x^2(x-1) - (x^3 - 1)}{(x-1)^2}$

7

$f(x) = -3x^2 + 5\sqrt{x}$ $f(x) = -3x^2 + 5x^{\frac{1}{2}}$ $f'(x) = -6x + \frac{5}{2}x^{-\frac{1}{2}}$	$\checkmark 5x^{\frac{1}{2}}$ $\checkmark -6x$ $\checkmark \frac{5}{2}x^{-\frac{1}{2}}$	(3)
$p(x) = \left(\frac{1}{x^3} + 4x\right)^2$ $= \frac{1}{x^6} + \frac{8}{x^2} + 16x^2$ $= x^{-6} + 8x^{-2} + 16x^2$ $p'(x) = -6x^{-7} - 16x^{-3} + 32x$	$\checkmark \frac{1}{x^6} + \frac{8}{x^2} + 16x^2$ $\checkmark x^{-6} + 8x^{-2} + 16x^2$ $\checkmark \checkmark \text{ answer/antwoord}$	(4)
<p>OR/OF</p> $p(x) = (x^{-3} + 4x)^2$ <p>by making use of the chain rule :</p> $p'(x) = 2(x^{-3} + 4x)(-3x^{-4} + 4)$ $p'(x) = -6x^{-7} - 16x^{-3} + 32x$		
	$\checkmark \checkmark 2(x^{-3} + 4x)$ $\checkmark \checkmark (-3x^{-4} + 4)$	(4)

7. ANSWERS TO FINDING THE TANGENT

1. $y = 2x^2$
 $m = 2\left(-\frac{1}{2}\right)^2$
 $m = \frac{1}{4}\checkmark$
 $y = \left(-\frac{1}{2}\right)^3$
 $y = -\frac{1}{8}\checkmark$
 $y + \frac{1}{8} = \frac{1}{4}\left(x + \frac{1}{2}\right)$
 $y = \frac{1}{4}x + \frac{1}{8}$
 $y = \frac{1}{4}x\checkmark$ (3)
2. $f'(x) = 3x^2 - 12x = 9$
 $f'(4) = 3(4)^2 - 12(4) = 9$
 $m = 9\checkmark$
 $f(4) = (4)^3 - 6(4)^2 - 12(4) = 9$
 $f(4) = 4\checkmark$
 $y - 4 = 9(x - 4)$
 $y - 4 = 9x - 36$
 $y = 9x - 32\checkmark$ (3)
3. $f'(x) = 3x^2 - 4x - 1$
 $m = f'(1) = 3(1)^3 - 4(1) - 1$
 $m = -2\checkmark$
 $f'(1) = (1)^3 - 2(1)^2 - (1) + 2$
 $f'(1) = 0\checkmark$
 $y = -2(x - 1)$
 $y = -2x + 2\checkmark$ (3)
4. $g'(x) = 3ax^2 + 6x + b$
 $g''(x) = 6ax + 6\checkmark$
 $g''(-1) = 0\checkmark$
 $-6a + 6 = 0$
 $a = 1\checkmark$
 $g(x)$ will be concave up for $x > -1\checkmark$ (4)

5.

<p>Substitute (2 ; b) in $y = x + \frac{12}{x}$</p> $b = 2 + \frac{12}{2}$ $b = 8$ <p>$m_{\text{tangent}} = \frac{dy}{dx}$</p> $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ $m_{\text{tangent}} = 1 - \frac{12}{2^2} = -2$ <p>$m_{\text{perp}} = \frac{1}{2}$</p> <p>Equation of perpendicular line:</p> $y - y_1 = m(x - x_1) \quad \text{OR} \quad y = mx + c$ $y - 8 = \frac{1}{2}(x - 2) \quad \quad 8 = \frac{1}{2}(2) + c$ $y = \frac{1}{2}x + 7 \quad \quad c = 7$ $y = \frac{1}{2}x + 7$	<p>✓ value of b</p> <p>✓ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$</p> <p>✓ gradient of perpendicular line</p> <p>✓ equation (4)</p>
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6.

<p>$f(x) = 2x^3 - 2x^2 + 4x - 1$</p> <p>$f'(x) = 6x^2 - 4x + 4$</p> <p>$f''(x) = 12x - 4$</p> <p>f is concave up when/is <i>konkaaf op</i> as $f''(x) > 0$</p> <p>$\therefore 12x - 4 > 0$</p> <p>$12x > 4$</p> <p>$x > \frac{1}{3}$</p>	<p>✓ first derivative/<i>eerste afgeleide</i></p> <p>✓ second derivative/<i>tweede afgeleide</i></p> <p>✓ $f''(x) > 0$</p> <p>✓ $x > \frac{1}{3}$ (4)</p>
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7.

$f(x) = x^3 + ax^2 + bx + 18$ $f'(x) = 3x^2 + 2ax + b$ At $x = 1$, $m_{\text{tan}} = -8$ $f'(1) = -8$ $3(1)^2 + 2a(1) + b = -8$ $3 + 2a + b = -8$ $2a + b = -11 \dots\dots\dots(1)$ $y = f(1)$ $= g(1)$ $= -8(1) + 20$ $= 12$ $1 + a + b + 18 = 12$ $a + b = -7 \dots\dots\dots(2)$ $a = -4$ $b = -3$	$\checkmark 3x^2 + 2ax + b$ $\checkmark f'(1) = -8$ or $3(1)^2 + 2a(1) + b = -8$ $\checkmark 1 + a + b + 18 = 12$ $\checkmark a = -4$ $\checkmark b = -3$
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(5)

Capricorn North
District
Mathematics
Calculus
Grade 12
Manual marking Guide
Level 3 & 4

17.ANSWERS TO COGNITIVE LEVEL 3 & 4

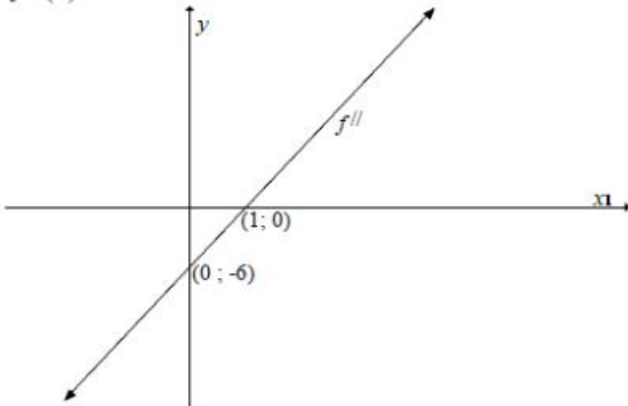
17.1. ACTIVITY 1

1.1	$-1 < x < 2$	✓✓ answer (2)
1.2	$x = \frac{-1+2}{2}$ $x = \frac{1}{2}$ <div>Answer Only: Full Marks</div>	✓ method ✓ answer (2)
1.3	From the graph $x > \frac{1}{2}$ <div>Answer Only: Full Marks</div>	✓✓ answer (2)
1.4	$g(x) = ax^3 + bx^2 + cx$ $g'(x) = 3ax^2 + 2bx + c = -6x^2 + 6x + 12$ $3a = -6, \quad 2b = 6, \quad c = 12$ $a = -2, \quad b = 3$ $g(x) = -2x^3 + 3x^2 + 12x$	✓ $g'(x) = 3ax^2 + 2bx + c$ ✓ $a = -2$ ✓ $b = 3$ ✓ $g(x) = -2x^3 + 3x^2 + 12x$ (4)
1.5	$g'\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right) + 12$ $m = \frac{27}{2} \quad \text{or } 13,5$ $y = -2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 + 12\left(\frac{1}{2}\right)$ $y = \frac{13}{2} \quad \text{or } 6,5$ $y - y_1 = m(x - x_1)$ $y - 6,5 = 13,5(x - 0,5)$ $y = 13,5x - 0,25$	✓ max gradient at $x = \frac{1}{2}$ ✓ answer ✓ y value ✓ substitution ✓ answer (5)
		[15]

17.2. ACTIVITY 2

2.1.1	$g(x) = (x+5)(x-x_1)^2$ $20 = 5(x_1)^2$ $x_1^2 = 4$ $x_1 = 2$ $g(x) = (x+5)(x-2)^2$ $g(x) = (x+5)(x^2 - 4x + 4)$ $g(x) = x^3 + x^2 - 16x + 20$	$\checkmark (x+5)$ \checkmark repeated root $\checkmark x_1 = 2$ $\checkmark g(x) = (x+5)(x^2 - 4x + 4)$ (4)
2.1.2	$g(x) = x^3 + x^2 - 16x + 20$ $g'(x) = 3x^2 + 2x - 16$ $3x^2 + 2x - 16 = 0$ $(3x+8)(x-2) = 0$ $x = -\frac{8}{3}$ or $x = 2$ $R\left(-\frac{8}{3}; \frac{1372}{27}\right)$ or $R(-2,67;50,81)$ $P(2;0)$	\checkmark derivative \checkmark equating to zero \checkmark factors \checkmark co-ordinates of R \checkmark co-ordinates of P (5)
2.1.3	$g''(x) = 6x + 2$ $g''(0) = 2$ \therefore concave up OR/OF $g''(x) = 6x + 2$ $6x + 2 = 0$ $x = -\frac{1}{3}$ is the point of inflection \therefore concave up	$\checkmark g''(x) = 6x + 2$ $\checkmark g''(0) = 2$ \checkmark conclusion (3) OR/OF $\checkmark g''(x) = 6x + 2$ $\checkmark x = -\frac{1}{3}$ \checkmark conclusion (3)

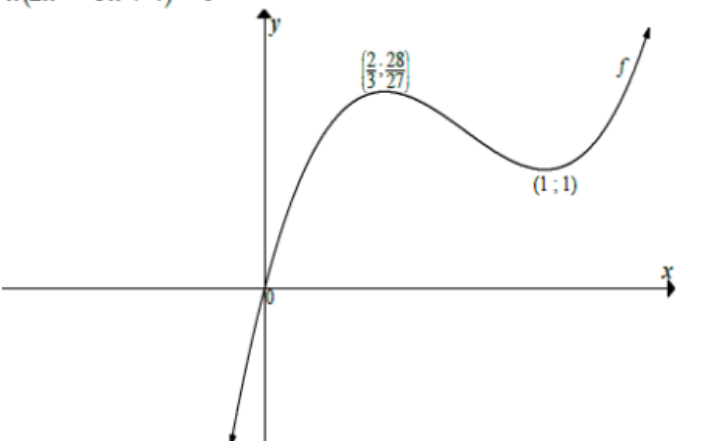
17.3. ACTIVITY 3

3.1	$f(x) = (x+2)(x-1)(x-4)$ $= (x^2 + x - 2)(x-4)$ $= x^3 + x^2 - 2x - 4x^2 - 4x + 8$ $= x^3 - 3x^2 - 6x + 8$ $b = -3 ; c = -6 ; d = 8$	$\checkmark \checkmark f(x) = (x+2)(x-1)(x-4)$ $\checkmark \text{ expansion}$ $\checkmark x^3 - 3x^2 - 6x + 8$ <p style="text-align: right;">(4)</p>
3.2	$f(x) = x^3 - 3x^2 - 6x + 8$ $f'(x) = 0$ $3x^2 - 6x - 6 = 0$ $x^2 - 2x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)}$ $= \frac{2 \pm \sqrt{12}}{2}$ $x = -0,73$	$\checkmark f'(x) = 0$ $\checkmark 3x^2 - 6x - 6$ $\checkmark \text{ substitution into correct formula}$ $\checkmark x = -0,73$ <p style="text-align: right;">(4)</p>
3.3	$f(x) = x^3 - 3x^2 - 6x + 8$ $f(-1) = (-1)^3 - 3(-1)^2 - 6(-1) + 8 \quad \text{or} \quad f(-1) = (1)(-2)(-5)$ $= 10 \quad \quad \quad = 10$ $f'(-1) = 3(-1)^2 - 6(-1) - 6$ $= 3$ $y - 10 = 3(x + 1)$ $y = 3x + 13$	$\checkmark f(-1) = 10$ $\checkmark f'(-1) = 3$ $\checkmark \text{ substitution}$ $\checkmark y = 3x + 13$ <p style="text-align: right;">(4)</p>
3.4	$f''(x) = 6x - 6$ 	$\checkmark f''(x) = 6x - 6$ $\checkmark x\text{- intercept}$ $\checkmark y\text{- intercept}$ <p style="text-align: right;">(3)</p>

3.5	f concave upwards $f''(x) > 0$ $6x - 6 > 0$ $x > 1$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> NOTE: Answer only 2 / 2 </div>	$\checkmark f''(x) > 0$ $\checkmark x > 1$	(2) [17]
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17.4. ACTIVITY4

4.1	$f'(x) = 0$ $6x^2 - 10x + 4 = 0$ $3x^2 - 5x + 2 = 0$ $(3x - 2)(x - 1) = 0$ $x = \frac{2}{3}$ or $x = 1$ $y = 2\left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)$ $y = 2(1)^3 - 5(1)^2 + 4(1)$ $y = \frac{28}{27}$ or $y = 1$ Turning points are $\left(\frac{2}{3}, \frac{28}{27}\right)$ and $(1; 1)$	\checkmark derivative \checkmark derivative = 0 \checkmark factors \checkmark x-values \checkmark y-values	(5)
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4.2	$2x^3 - 5x^2 + 4x = 0$ $x(2x^2 - 5x + 4) = 0$ $x = 0 \quad \text{or} \quad x = \frac{5 \pm \sqrt{25 - 4(2)(4)}}{4}$ $= \frac{5 \pm \sqrt{-7}}{4}$ <p>No real roots / <i>Geen reële wortels</i></p> <p>OR / OF</p> $2x^3 - 5x^2 + 4x = 0$ $x(2x^2 - 5x + 4) = 0$ $x = 0 \quad \text{or} \quad b^2 - 4ac = 25 - 4(2)(4)$ $= -7 < 0$ <p>No real roots / <i>Geen reële wortels</i></p>	$\checkmark x(2x^2 - 5x + 4) = 0$ $\checkmark x = 0$ $\checkmark \frac{5 \pm \sqrt{-7}}{4}$ <p>(3)</p> $\checkmark x(2x^2 - 5x + 4) = 0$ $\checkmark x = 0$ $\checkmark b^2 - 4ac < 0$ <p>(3)</p>
4.3	$f(x) = 2x^3 - 5x^2 + 4x$ $x(2x^2 - 5x + 4) = 0$ 	$\checkmark (0 ; 0)$ $\checkmark \text{turning points}$ $\checkmark \text{shape}$ <p>(3)</p>

17.5. ACTIVITY 5

5.1	$f(x) = x^3 - 6x^2 + 9x$ $f'(x) = 3x^2 - 12x + 9$ $f''(x) = 6x - 12 = 0$ $x = 2$ $f''(0) = 6(0) - 12 = -12$ $f''(3) = 6(3) - 12 = 6$ <div style="text-align: center;"> </div> <p>Point of inflection at $x = 2$</p>	$\checkmark x^3 - 6x^2 + 9x$ $\checkmark 3x^2 - 12x + 9$ $\checkmark 6x - 12$ $\checkmark 6x - 12 = 0$ \checkmark explanation	(5)
5.2		\checkmark shape $\checkmark (0; 0)$ $\checkmark (3; 0)$ as TP $\checkmark (1; 4)$	(4)
5.3	f concave up for $x > 2$ $y = -f(x)$ will be concave down for $x > 2$	$\checkmark \checkmark x > 2$	(2)
5.4.1	$(3; 7)$	$\checkmark 3$ $\checkmark 7$	(2)
5.4.2	<p>Do not agree with Claire as her statement is incorrect. Between $x = 1$ and $x = 3$ the graph of f is decreasing. Therefore at $x = 2$ the gradient will have a negative value.</p> <p><i>Stem nie saam met Claire nie, want haar stelling in verkeerd. Die grafiek van f is dalend/afnemend tussen $x = 1$ en $x = 3$. By $x = 2$ moet die gradiënt dus 'n negatiewe waarde hê.</i></p> <p>OR/OF</p> $f'(2) = 3(2)^2 - 12(2) + 9$ $= -3$ $\neq 1$	\checkmark no \checkmark justification	(2)

[15]

17.6. ACTIVITY 6

6.1	$f'(x) = 3x^2 + 8x - 3 = 0$ $(3x-1)(x+3) = 0$ $x = \frac{1}{3} \text{ or } x = -3$	✓ equating derivative to zero ✓ factors ✓ x - values (3)
6.2	$f''(x) = 6x + 8$ $6x + 8 < 0$ $x < -\frac{4}{3}$ OR $x = \frac{\frac{1}{3} - 3}{2}$ $= \frac{4}{3}$ $\therefore x < -\frac{4}{3}$	✓ $6x + 8$ ✓✓ $x < -\frac{4}{3}$ ✓ $\frac{\frac{1}{3} - 3}{2}$ ✓✓ $x < -\frac{4}{3}$ (3)
6.3	$x \leq -3 \text{ or } x \geq \frac{1}{3}$ OR/OF $[-\infty; -3] \cup \left[\frac{1}{3}; \infty\right]$	✓ $x \leq -3$ ✓ $x \geq \frac{1}{3}$ ✓ $[-\infty; -3]$ ✓ $\left[\frac{1}{3}; \infty\right]$ (2) (2)
6.4	$f(0) = -18$ $d = -18$ $f(x) = ax^3 + bx^2 + cx - 18$ $f'(x) = 3ax^2 + 2bx + c$ $f'(x) = 3x^2 + 8x - 3$ $3a = 3 \quad 2b = 8$ $a = 1 \quad b = 4 \quad c = -3$ $f(x) = x^3 + 4x^2 - 3x - 18$ OR/OF $f'(x) = 3x^2 + 8x - 3$ By integration/Deur integrasie $f(x) = x^3 + 4x^2 - 3x + d$ $f(0) = d = -18$ $a = 1$ $b = 4$ $c = -3$	✓ $d = -18$ ✓ $f'(x) = 3ax^2 + 2bx + c$ ✓ $a = 1$ ✓ $b = 4$ ✓ $c = -3$ (5) ✓ $f(x) = x^3 + 4x^2 - 3x + d$ ✓ $d = -18$ ✓ $a = 1$ ✓ $b = 4$ ✓ $c = -3$ (5)
		[13]

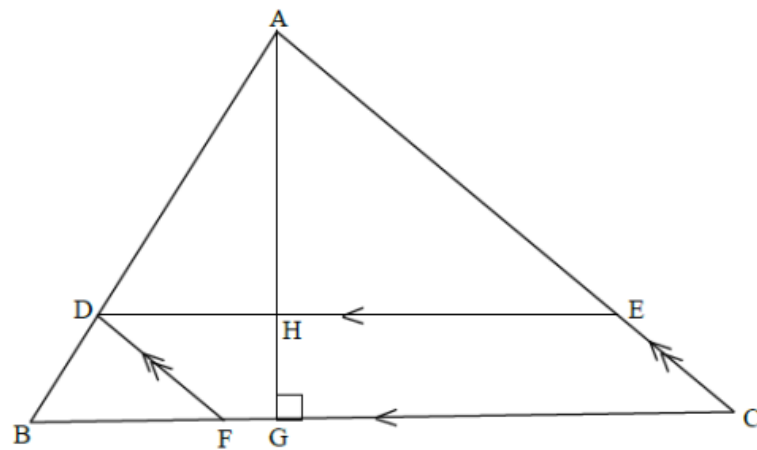
17.7. ACTIVITY 7

7.1	<p>Total surface area = $2\ell w + 2wh + 2\ell h$ but: $\ell = 3w$ Total surface area = $6w^2 + 2wh + 6wh$</p> <p>$C = 15(6w^2) + 6(2wh + 6wh)$ $= 15(6w^2) + 6(8wh)$ $= 90w^2 + 48wh$</p>	<p>✓ $2\ell w + 2wh + 2\ell h$ ✓ $\ell = 3w$</p> <p>✓ $15(6w^2)$ ✓ $6(2wh + 6wh)$</p>
7.2	<p>$5 = 3w^2h$ $h = \frac{5}{3w^2}$ $C = 90w^2 + 48wh$ $C(w) = 90w^2 + 48w\left(\frac{5}{3w^2}\right)$ $= 90w^2 + 80w^{-1}$ $C'(w) = 180w - 80w^{-2}$ $180w - 80w^{-2} = 0$ $180w^3 - 80 = 0$ $w^3 = \frac{80}{180}$ $w = \sqrt[3]{\frac{80}{180}}$ $w = 0,76$</p>	<p>✓ $h = \frac{5}{3w^2}$</p> <p>✓ substitution</p> <p>✓ $C(w) = 90w^2 + 80w^{-1}$ ✓ derivative ✓ equating derivative to zero</p> <p>✓ value of w</p>
		(4)
		(6)
		[10]

17.8. ACTIVITY 8

8.1	36cm	✓ answer (1)
8.2	<p>$\therefore t = 6$ ($-2t^2 + 3t - 6$) have no real roots Insect reaches the floor only once.</p>	<p>✓✓✓ only once (3)</p>
8.3	<p>$h(t) = -2t^3 + 15t^2 - 24t + 36$ $h'(t) = -6t^2 + 30t - 24$ $-6t^2 + 30t - 24 = 0$ $t^2 - 5t + 4 = 0$ $(t-4)(t-1) = 0$ $t = 4$ or $t = 1$ Only $t = 4$ because maximum value required $h = -2(4)^3 + 15(4)^2 - 24(4) + 36 = 52 \text{ cm}$</p>	<p>✓ expansion</p> <p>✓ $-6t^2 + 30t - 24 = 0$</p> <p>✓ both values</p> <p>✓ answer (4)</p>
		[8]

17.9. ACTIVITY 9



9.1	$\frac{AH}{HG} = \frac{3}{2}$	✓ answer (1)
9.2	<p>Area of a parallelogram = base \times \perp height</p> <p>Area = $\frac{3}{5}(5-t) \cdot \frac{2}{5}t$</p> <p>Area = $\frac{6}{25}(5-t)t$</p> <p>$A(t) = -\frac{6}{25}t^2 + \frac{6}{5}t$</p> <p>$A'(t) = -\frac{12}{25}t + \frac{6}{5}$</p> <p>$-\frac{12}{25}t + \frac{6}{5} = 0$</p> <p>$12t - 30 = 0$</p> <p>$t = \frac{30}{12}$ or $\frac{5}{2}$</p>	<p>✓ $\frac{2}{5}t$</p> <p>✓ $\frac{3}{5}(5-t)$</p> <p>✓ $A(t) = -\frac{6}{25}t^2 + \frac{6}{5}t$</p> <p>✓ $-\frac{12}{25}t + \frac{6}{5}$</p> <p>✓ answer (5)</p>
		[6]

ACTIVITY 10

$y = x^2 + 2$ $P(x; x^2 + 2)$ $B(0; 3)$ $PB^2 = (x - 0)^2 + (x^2 + 2 - 3)^2$ $= x^2 + x^4 - 2x^2 + 1$ $= x^4 - x^2 + 1$ PB will be a minimum if PB^2 is a minimum $\frac{d(PB^2)}{dx} = 4x^3 - 2x$ $4x^3 - 2x = 0$ $x(2x^2 - 1) = 0$ $x = 0$ or $x^2 = \frac{1}{2}$ $x = \frac{1}{\sqrt{2}}$ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $= \frac{1}{4} - \frac{1}{2} + 1$ $= \frac{3}{4}$ $PB = \frac{\sqrt{3}}{2} = 0,87$ OR/OF	$\checkmark (x - 0)^2 + (x^2 + 2 - 3)^2$ $\checkmark x^4 - x^2 + 1$ $\checkmark 4x^3 - 2x$ $\checkmark \frac{d(PB^2)}{dx} = 0$ $\checkmark x = \frac{1}{\sqrt{2}}$ $\checkmark PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ \checkmark answer OR/OF
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<p>Gradient of tangent to curve = $2x$</p> <p>Gradient of line joining B and the curve = $\frac{x^2+2-3}{x-0}$ $= \frac{x^2-1}{x}$</p> <p>Shortest distance will be where tangent to curve is perpendicular to the line joining P and the curve.</p> $\frac{x^2-1}{x} = -\frac{1}{2x}$ $2x(x^2-1) = -x$ $2x^3 - 2x = 0$ $x(2x^2-1) = 0$ $x = 0 \text{ or } x^2 = \frac{1}{2}$ $x = \frac{1}{\sqrt{2}}$ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $= \frac{1}{4} - \frac{1}{2} + 1$ $= \frac{3}{4}$ $PB = \frac{\sqrt{3}}{2} = 0,87$ <p>OR/OF</p> <p>$P(k; k^2 + 2)$ and $B(0; 3)$</p> <p>$BP \perp$ tangent passing through $y = x^2 + 2$ at P.</p> $m_{\text{tangent at P}} = 2k$ $m_{BP} = -\frac{1}{2k}$ <p>Equation of BP: $y = \left(-\frac{1}{2k}\right)x + 3$</p> $y_P = \left(-\frac{1}{2k}\right)(k) + 3 = 2,5$ $\Rightarrow k^2 + 2 = 2,5 \text{ and so } k = \sqrt{0,5} \text{ and } P(\sqrt{0,5}; 2,5)$ $BP = \sqrt{(\sqrt{0,5} - 0)^2 + (2,5 - 3)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} = 0,87$	<p>✓ = $2x$</p> <p>✓ = $\frac{x^2-1}{x}$</p> <p>✓ $\frac{x^2-1}{x} = -\frac{1}{2x}$</p> <p>✓ $2x^3 - 2x = 0$</p> <p>✓ $x = \frac{1}{\sqrt{2}}$</p> <p>✓ $PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$</p> <p>✓ answer</p> <p>OR/OF</p> <p>✓ $P(k; k^2 + 2)$</p> <p>✓ $m_{\text{tangent at P}} = 2k$</p> <p>✓ $m_{BP} = -\frac{1}{2k}$</p> <p>✓ $y = \left(-\frac{1}{2k}\right)x + 3$</p> <p>✓ value of y at P</p> <p>✓ value of k</p> <p>✓ answer</p>
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[7]

SOLUTIONS FOR COGNITIVE LEVEL 1 AND 2 ACTIVITIES

INDEPENDENT EVENTS, MUTUALLY EXCLUSIVE EVENTS AND VENN DIAGRAMS

QUESTION 1

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$0,428 = 0,12 + 0,35 - P(A \text{ and } B)\checkmark$$

$$\therefore P(A \text{ and } B) = 0,042\checkmark$$

hence for independent events:

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$0,042 = 0,12 \times 0,35\checkmark$$

$$0,042 = 0,042$$

Thus, A and B are independent events \checkmark

QUESTION 2

for independent events:

$$P(A \text{ and } B) = P(A) \times P(B)\checkmark$$

$$0,126 = 0,2 \times 0,63\checkmark$$

$$0,126 = 0,126$$

Thus, A and B are independent events \checkmark

QUESTION 3

1.

for independent events:

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ and } B) = 0,4 \times 0,5\checkmark$$

$$P(A \text{ and } B) = 0,2\checkmark$$

2.

identity rule (basic rule of probability):

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = 0,4 + 0,5 - 0,2\checkmark$$

$$P(A \text{ or } B) = 0,7\checkmark$$

3.

for complementary rule:

$$P(\text{not } A \text{ and not } B) = 1 - P(A \text{ or } B)\checkmark$$

$$P(\text{not } A \text{ and not } B) = 1 - 0,7\checkmark$$

$$P(\text{not } A \text{ and not } B) = 0,3\checkmark$$

QUESTION 4

1.

for independent events:

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$0,12 = 0,4 \times P(B)\checkmark$$

$$P(B)=0,3\checkmark$$

2.

identity rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)\checkmark$$

$$P(A \text{ or } B) = 0,4 + 0,3 - 0,12$$

$$P(A \text{ or } B) = \frac{29}{50}$$

$$P(A \text{ or } B) = 0,58\checkmark$$

3.

complementary rule:

$$P(\text{not } A \text{ and not } B) = 1 - P(A \text{ or } B)\checkmark$$

$$P(\text{not } A \text{ and not } B) = 1 - 0,58\checkmark$$

$$P(\text{not } A \text{ and not } B) = 0,42\checkmark$$

QUESTION 5

for mutually exclusive events:

$$P(A \text{ or } B) = P(A) + P(B)\checkmark$$

$$0,57 = P(A) + 2P(A)$$

$$\therefore 3P(A) = 0,57$$

$$\Rightarrow P(A) = 0,19\checkmark$$

Now:

$$P(B) = 2P(A)$$

$$P(B) = 2(0,19)$$

$$P(B) = 0,38\checkmark$$

QUESTION 6

for mutually exclusive events:

$$P(A \text{ or } B) = P(A) + P(B)$$

$$0,96 = P(A) + 3P(A)$$

$$\therefore 4P(A) = 0,96$$

$$\Rightarrow P(A) = \frac{0,96}{4}$$

$$\Rightarrow P(A) = 0,24$$

$$P(B') = 1 - P(B)$$

$$0,28 = 1 - P(B)$$

$$\therefore P(B) = 1 - 0,28$$

$$\Rightarrow P(B) = 0,72$$

$$\text{hence } P(A \text{ or } B) = P(A) + P(B)$$

$$0,96 = 0,24 + 0,72$$

$$0,96 = 0,96$$

Therefore, A and B are Mutually exclusive.

QUESTION 7

1.

Complementary rule

$$P(A) = 1 - P(\text{not } A)$$

$$P(A) = 1 - 0,45$$

$$P(A) = 0,55\checkmark$$

2.

for mutually exclusive events:

$$P(A \text{ or } B) = P(A) + P(B)\checkmark$$

$$P(A \text{ or } B) = 0,55 + 0,35$$

$$P(A \text{ or } B) = 0,9\checkmark$$

3.

For independent events:

$$P(A \text{ and } B) = P(A) \times P(B)\checkmark$$

$$P(A \text{ and } B) = 0,55 \times 0,35$$

$$P(A \text{ and } B) = 0,1925$$

$$P(A \text{ and } B) = 0,19\checkmark$$

QUESTION 8

1.

for mutually exclusive events:

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \text{ or } B) = \frac{3}{8} + \frac{1}{4}$$

$$P(A \text{ or } B) = \frac{5}{8}$$

2.

identity rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = \frac{3}{8} + \frac{1}{4} - P(A \text{ and } B)$$

but

for independent events:

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(A \text{ and } B) = \frac{3}{8} \times \frac{1}{4}$$

$$P(A \text{ and } B) = \frac{3}{32}$$

$$\text{hence, } P(A \text{ or } B) = \frac{5}{8} - \frac{3}{32}$$

$$P(A \text{ and } B) = \frac{3}{32}$$

QUESTION 9

1.

for mutually exclusive events:

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \text{ or } B) = 0,2 + 0,7$$

$$P(A \text{ or } B) = 0,9$$

2.

identity rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

but, A and B are independent: i. e. $P(A \text{ and } B) = P(A) \times P(B)$

$$\Rightarrow P(A \text{ or } B) = P(A) + P(B) - P(A) \times P(B)$$

$$P(A \text{ or } B) = 0,2 + 0,7 - 0,2 \times 0,7$$

$$P(A \text{ or } B) = 0,9 - 0,14$$

$$P(A \text{ or } B) = 0,76$$

QUESTION 10

for independent events:

$$P(A \text{ and } B) = P(A) \times P(B)\checkmark$$

$$0,2 = (x + 0,2) \times (0,2 + 0,3)$$

$$\therefore x = 0,2\checkmark$$

for the y value:

$$x + 0,2 + 0,3 + y = 1\checkmark$$

$$0,2 + 0,2 + 0,3 + y = 1$$

$$y = 1 - 0,7$$

$$y = 0,3\checkmark$$

QUESTION 11

for independent events:

$$P(A \text{ and } B) = P(A) \times P(B)\checkmark$$

$$0,1 = (x + 0,1) \times 0,4\checkmark$$

$$0,1 = 0,4x + 0,04$$

$$0,4x = 0,06$$

$$x = 0,15\checkmark$$

for the value of y:

$$x + 0,1 + 0,3 + y = 1\checkmark$$

$$0,15 + 0,1 + 0,3 + y = 1$$

$$y = 1 - 0,55$$

$$y = 0,45\checkmark$$

QUESTION 12

1.

Venn diagram should include the following information:

$$n(S) = 600$$

$$n(H) = 372$$

$$n(R) = 288$$

$$n(\text{neither } H \text{ or } R) = 56$$

$$n(H \text{ and } R) = x$$

2.

$$(372 - x) + x + (288 - x) + 56 = 600\checkmark$$

$$716 - x = 600$$

$$x = 116\checkmark$$

3.

for mutually exclusive events: $P(A \text{ and } B) = 0$

but $P(A \text{ and } B) \neq 0$ in this case✓

∴ No✓, A and B are not mutually exclusive events.

QUESTION 13

1. Venn diagram must include the following information:

$$nS=240$$

$$nR=122$$

$$nB=58$$

$$nC=96$$

$$nR \text{ or } B \text{ or } C=16$$

$$nR \text{ and } B=22$$

$$nC \text{ and } B=26$$

$$nR' \text{ or } B' \text{ or } C'=26$$

$$nR \text{ and } C=x$$

2.

$$100 - x + x + 70 - x + 6 + 16 + 10 + 26 + 26 = 240✓$$

$$254 - x = 240✓$$

$$-x = 240 - 254$$

$$-x = -14$$

$$x = 14✓$$

3.

3.1.

$$P(\text{only } B) = \frac{26}{240}✓$$

$$P(\text{only } B) = \frac{13}{120}$$

$$P(\text{only } B) = 0,11✓$$

3.2.

$$P(C') = 1 - \frac{96}{240}✓$$

$$P(C') = 1 - \frac{2}{5}$$

$$P(C') = \frac{3}{5}$$

$$P(C') = 0,6✓$$

3.3.

$$P(\text{at least two sports}) = \frac{16 + 6 + 14 + 10}{240} \checkmark$$

$$P(\text{at least two sports}) = \frac{46}{240}$$

$$P(\text{at least two sport}) = \frac{23}{120}$$

$$P(\text{at least two sport}) = 0,19 \checkmark$$

QUESTION 14

1.

11 students \checkmark

2.

Venn diagram must include the following information:

$$n(\text{Sample Space}) = 80$$

$$n(N) = 44$$

$$n(G) = 33$$

$$n(L) = 39$$

$$n(N \text{ and } L) = 23$$

$$n(G \text{ and } L) = 19$$

$$n(N \text{ and } G \text{ and } L) = 9$$

$$n(N \text{ or } G \text{ or } L) = 69$$

3.

$$21 - x + x + 14 - x + 9 + 14 + 10 + 6 + 11 = 80 \checkmark \checkmark$$

$$85 - x = 80 \checkmark$$

$$x = 5$$

4.

$$P(\text{at least two magazines}) = \frac{5 + 14 + 10 + 9}{80} \checkmark \checkmark$$

$$P(\text{at least two magazines}) = 0,475 \checkmark$$

QUESTION 15

1.

Venn diagram must include the following information:

$$n(\text{Sample Space}) = 173$$

$$n(M) = 110$$

$$n(F) = 55$$

$$n(S) = 67$$

$$n(M \text{ and } F \text{ but } S') = 20$$

$$n(M \text{ and } S \text{ but } F') = 11$$

$$n(F \text{ and } S \text{ but } M') = 16$$

$$n(M \text{ and } f \text{ and } S) = x$$

2.

$$79 - x + 20 + x + 11 + 19 - x + 16 + 40 - x = 173\checkmark$$

$$185 - 2x = 173\checkmark$$

$$x = 6$$

QUESTION 16

1.

Venn diagram must include the following information:

$$n(\text{Sample Space}) = 103$$

$$n(C \text{ or } F \text{ or } V)' = 2$$

$$n(\text{only } V) = 5$$

$$n(\text{only } C) = 2$$

$$n(F') = 21$$

$$n(\text{only } F) = 3$$

$$n(C \text{ and } F) = 66$$

$$n(V \text{ and } F) = 75$$

$$n(C \text{ and } V \text{ and } F) = x$$

2.

$$5 + 12 + 2 + x + 75 - x + 66 - x + 3 + 2 = 103\checkmark$$

$$x = 62\checkmark$$

3.

$$P(\text{only eats chicken and fish and no vegetables}) = \frac{4}{103}\checkmark\checkmark$$

$$P(\text{ANY TWO}) = \frac{12 + 4 + 13}{103}\checkmark$$

$$P(\text{ANY TWO}) = \frac{29}{103}\checkmark$$

QUESTION 17

1.

Venn diagram must include the following information:

$$n(\text{Sample Space}) = 2140$$

$$n(A) = 890$$

$$n(N) = 680$$

$$n(P \text{ and } A) = 120$$

$$n(P \text{ and } N) = 110$$

$$n(A \text{ and } N) = 140$$

$$n(P) = x$$

$$n(A \text{ or } N \text{ or } P) = 2140$$

2.

$$x + 80 + 40 + 70 + 470 + 100 + 670 = 2140 \checkmark$$

$$x + 1430 = 2140$$

$$x = 710 \checkmark$$

Number said lack of parental support

$$= 710 + 80 + 40 + 70$$

$$= 900$$

3.

$$P(\text{exactly 2 problems}) = \frac{80 + 100 + 70}{2140} \checkmark \checkmark$$

$$P(\text{exactly 2 problems}) = \frac{250}{2140}$$

$$P(\text{exactly 2 problems}) = 0,12 \checkmark (0,1168224299)$$

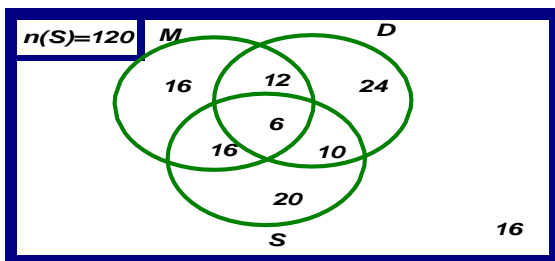
QUESTION 18

1.

$$n(\text{Sample Space}) = 120$$

$$n(M) = 50$$

$$n(S) = 52$$



$$n(D) = 52$$

$$n(M \text{ and } D) = 18$$

$$n(M \text{ and } S) = 22$$

$$n(S \text{ and } D) = 16$$

$$n(M \text{ and } S \text{ and } D) = 6$$

2.

$$P(\text{exactly one newspaper}) = \frac{n(\text{only } M \text{ or only } D \text{ or only } S)}{n(\text{Sample Space})}$$

$$P(\text{exactly one newspaper}) = \frac{16 + 24 + 20}{120}$$

$$P(\text{exactly one newspaper}) = \frac{60}{120}$$

$$P(\text{exactly one newspaper}) = \frac{1}{2} = 0,5$$

3.

$$P(M' \text{ or } D' \text{ or } S') = \frac{16}{120} \sqrt{2}$$

$$P(M' \text{ or } D' \text{ or } S') = \frac{2}{15}$$

$$P(M' \text{ or } D' \text{ or } S') = 0,13 \sqrt{2}$$

CONTINGENCY TABLE

QUESTION 1

1.

$$P(\text{boy, tennis or squash}) = \frac{18}{120} \times \frac{20}{120} \sqrt{2}$$

$$P(\text{boy, tennis or squash}) = \frac{19}{60}$$

$$P(\text{boy, tennis or squash}) = 0,32 \sqrt{2}$$

2.

$$P(\text{learner, tennis}) = \frac{41}{120}$$

$$P(\text{learner, tennis}) = 0,34 \sqrt{2}$$

3.

$$P(\text{girl}) = \frac{50}{120}$$

$$P(\text{girl}) = \frac{5}{12}$$

$$P(\text{girl}) = 0,42 \sqrt{2}$$

4.

for independent events:

$$P(\text{boy and golf}) = \frac{32}{32} \times P(\text{golf})$$

$$P(\text{boy and golf}) = \quad P(\text{boy and golf}) =$$

$$\frac{120}{4} \sqrt{\frac{4}{15}}$$

$$P(\text{boy and golf}) = 0,27$$

On the other hand

$$P(\text{boy}) \times P(\text{golf}) = \frac{70}{120} \times \frac{44}{120} \sqrt{}$$

$$P(\text{boy}) \times P(\text{golf}) = \frac{77}{360} \sqrt{}$$

$$P(\text{boy}) \times P(\text{golf}) = 0,21$$

$$\therefore 0,27 \neq 0,21$$

Events are not independent.

QUESTION 2

$$P(\text{Right and Light}) = P(\text{Right}) \times P(\text{Light}) \sqrt{}$$

$$\frac{a}{60} = \frac{48}{60} \times \frac{20}{50} \sqrt{}$$

$$\frac{a}{60} = \frac{4}{15}$$

$$a = 16 \sqrt{}$$

$$b = 20 - 16$$

$$b = 4 \sqrt{}$$

$$c = 48 - 16$$

$$c = 32 \sqrt{}$$

QUESTION 3

1.

$$P(\text{male}) = \frac{83}{180}$$

$$P(\text{male}) = 0,46 \sqrt{}$$

2.

$$P(\text{not game park}) = 1 - P(\text{game park})$$

$$P(\text{not game park}) = 1 - \frac{62}{180} \sqrt{}$$

$$59$$

$$P(\text{not game park}) = \frac{60}{60}$$

$$P(\text{not game park}) = 0,66$$

3.

events are independent if:

$$P(\text{male and home}) = P(\text{male}) \times P(\text{home}) \sqrt{}$$

$$13$$

$$\therefore P(\text{male and home}) = \frac{1}{180} \sqrt{}$$

$$P(\text{male and home}) = 0,07222$$

Whereas

$$P(\text{male}) \times P(\text{home}) = \frac{83}{180} \times \frac{20}{180}$$

$$P(\text{male}) \times P(\text{home}) = \frac{83}{1620}$$

$$P(\text{male}) \times P(\text{home}) = 0,05123\checkmark$$

$$\text{Therefore } P(\text{male and home}) \neq P(\text{male}) \times P(\text{home})$$

Thus, events are not independent✓

QUESTION 4

1.

$$a = 60 + 100$$

$$a = 160\checkmark$$

2.

$$P(M) = \frac{60}{160}\checkmark$$

$$P(M) = \frac{3}{8}$$

$$P(M) = 0,375\checkmark$$

3.

for independent events:

$$P(\text{Male and Coffee}) = P(\text{Male}) \times P(\text{Coffee})\checkmark$$

$$\frac{b}{160} = \frac{3}{8} \times \frac{80}{160}\checkmark\checkmark$$

$$16b = 480$$

$$b = 30\checkmark$$

SOLUTIONS FOR COGNITIVE LEVEL 3 AND 4

QUESTION 1

$x + \frac{1}{6} + \frac{5}{12} + \frac{1}{3} = 1\checkmark$ $x = \frac{1}{12}\checkmark$ $\therefore P(A) = \frac{1}{12} + \frac{1}{6}$ $P(A) = \frac{1}{4}\checkmark$

QUESTION 2

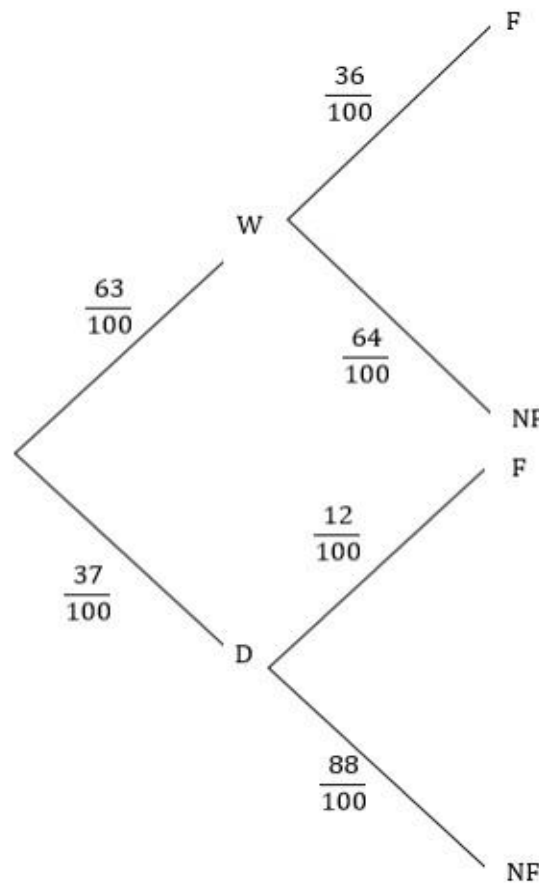
Disagree. The events A and B have an intersection and are therefore not mutually exclusive.
$P(B \text{ or } C) = P(B) + P(C)\checkmark$ $P(B \text{ or } C) = 0,3 + 0,32$ $P(B \text{ or } C) = 0,62\checkmark$
$P(A \text{ and } B) = P(A) \cdot P(B)$ $P(A \text{ and } B) = 0,45 \times 0,3$ $P(A \text{ and } B) = 0,135\checkmark$ $\therefore P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $P(A \text{ or } B) = 0,45 + 0,3 - 0,135$ $P(A \text{ or } B) = 0,62\checkmark \quad (0,615)$

QUESTION 3

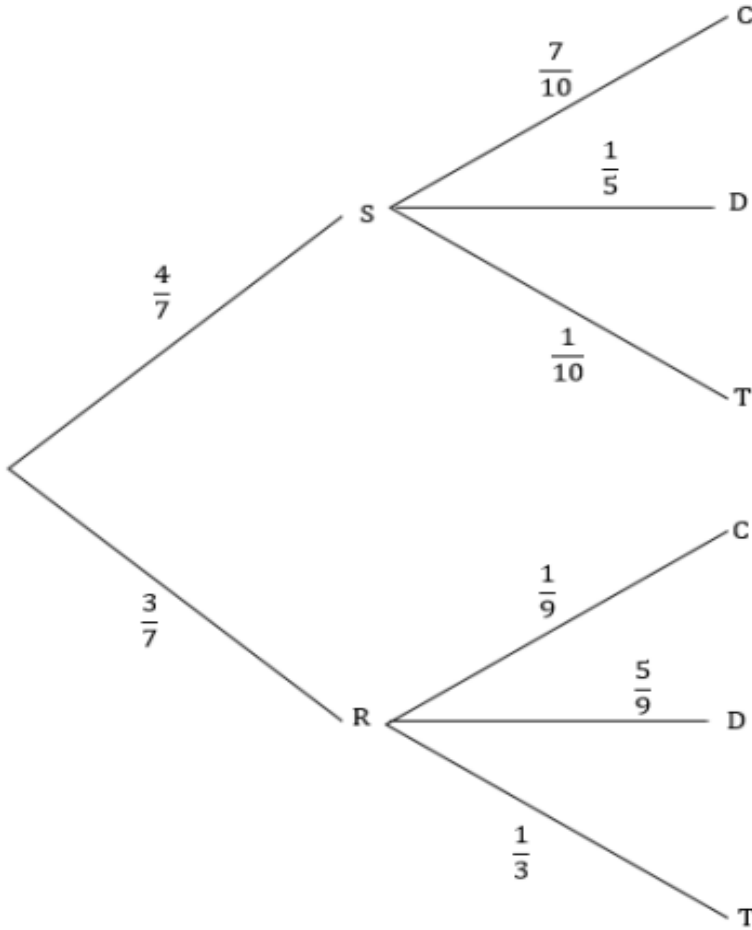
SOLUTION
$P(\text{pass Music or Art})$ $= P(\text{Pass Music}) + P(\text{pass Art})$ $- P(\text{pass Music and Art})\checkmark$ $P(\text{pass Music or Art}) = 0,4 + 0,6 - 0,3\checkmark$ $P(\text{pass Music or Art}) = 0,7\checkmark$

TREE DIAGRAMS

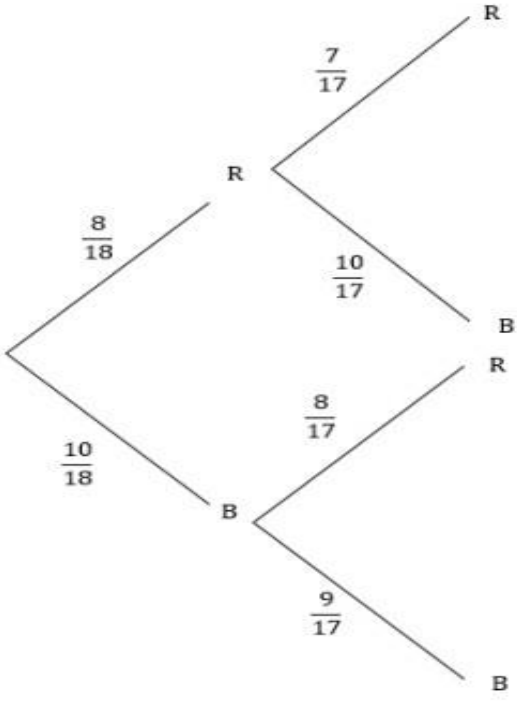
Activity 1

1.1	
1.2	$P(NF) = P(R \cap NF) + P(D \cap NF)$ $P(NF) = \frac{407}{1\,250} + \frac{252}{625}$ $P(NF) = 0,7288$
1.3	$P(D \cap F) = \frac{37}{100} \times \frac{12}{100}$ $P(D \cap F) = \frac{111}{2\,500}$ $P(D \cap F) = 0,044$

Activity 2

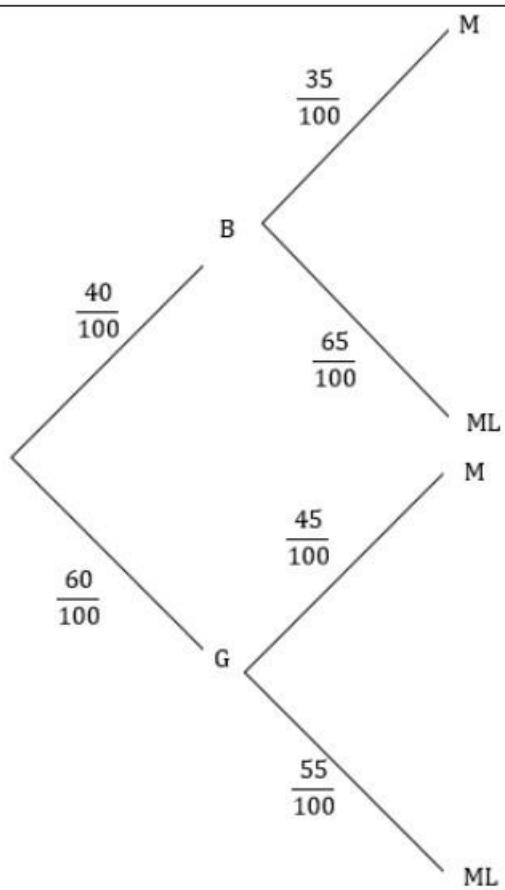
2.1	
2.2.1	$P(C) = P(S \cap C) + P(R \cap C)$ $P(C) = \frac{2}{5} + \frac{1}{21}$ $P(C) = 0,45$
2.2.2	$P(T) = P(S \cap T) + P(R \cap T)$ $P(T) = \frac{2}{35} + \frac{1}{7} = 0,3$
2.2.3	$\text{No. of days} = P(D) \times 245$ $\text{No. of days} = \left(\frac{4}{35} + \frac{5}{21}\right) \times 245 = 87 \text{ days}$

Activity 3

3.1	
3.2.1	$P(R \cap R) = \frac{28}{153}$ $P(R \cap R) = 0,183$
3.2.2	$P(\text{at least one blue}) = 1 - P(R \cap R)$ $P(\text{at least one blue}) = 1 - \frac{28}{153}$ $P(\text{at least one blue}) = \frac{125}{153}$ $P(\text{at least one blue}) = 0,817$
3.2.3	$P(R \cap B) = \frac{40}{153}$ $P(R \cap B) = 0,261$

Activity 4

4.1



$$P(M) = P(G \cap M) + P(B \cap M)$$

$$P(M) = \frac{117}{400} + \frac{7}{50}$$

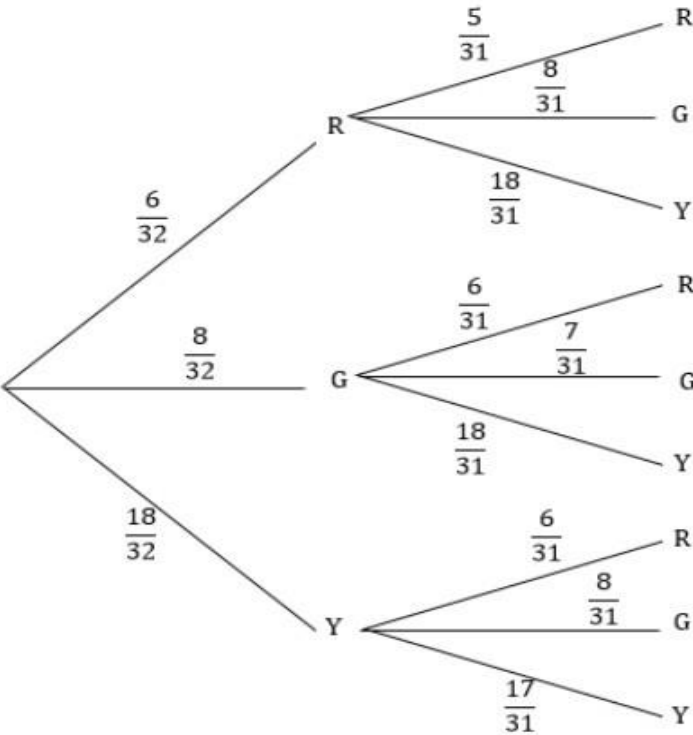
$$P(M) = \frac{173}{400}$$

$$P(M) = 0,433$$

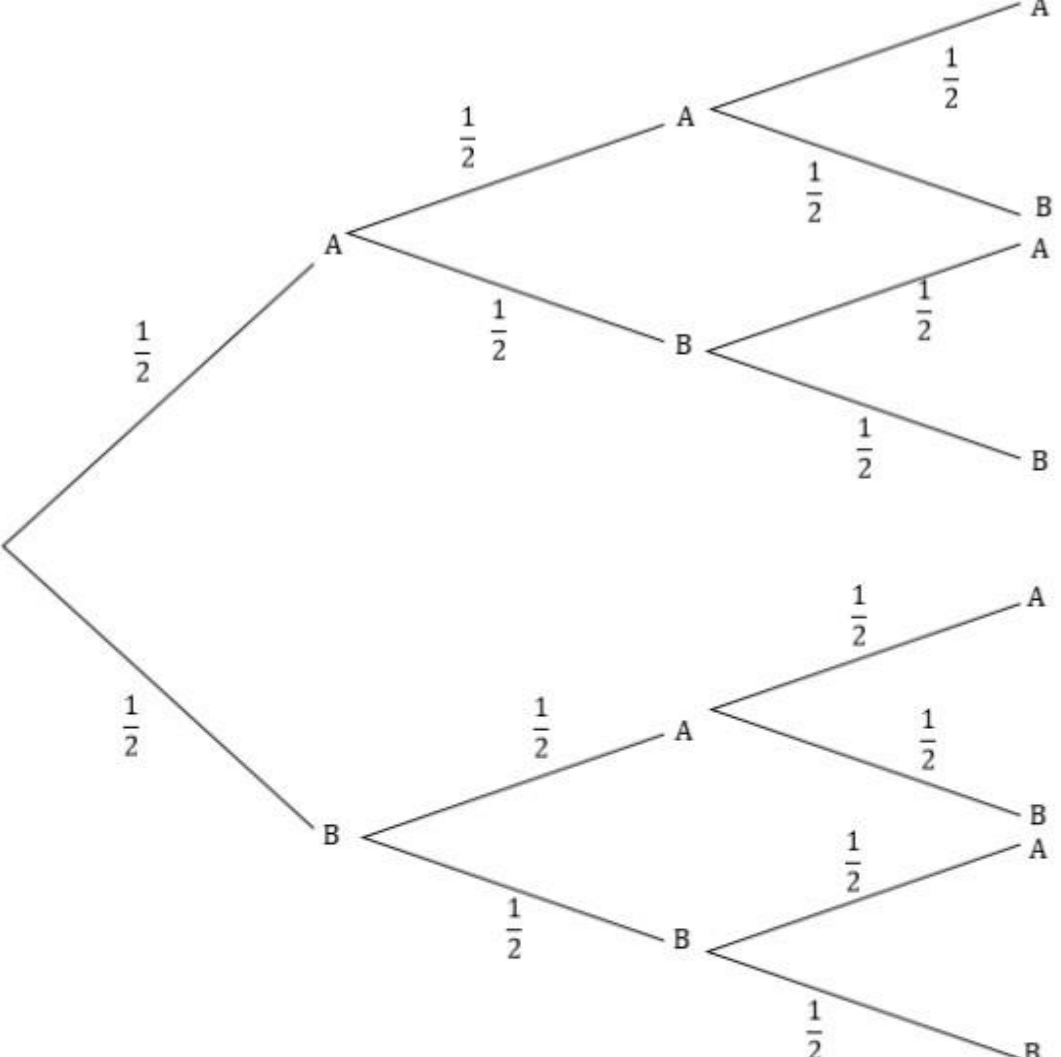
Activity 5

5.1	$P(B) = \frac{20}{35} = \frac{4}{7}$
5.2	
5.3	$P(B \cap G \cap B) = \frac{190}{1\,309}$ $P(B \cap G \cap B) = 0,145$
5.4	$P(G \cap G \cap G) = \frac{13}{187}$ $P(G \cap G \cap G) = 0,070$
5.5	$P(\text{at least one boy}) = 1 - P(G \cap G \cap G)$ $P(\text{at least one boy}) = 1 - \frac{13}{187} = 0,930$

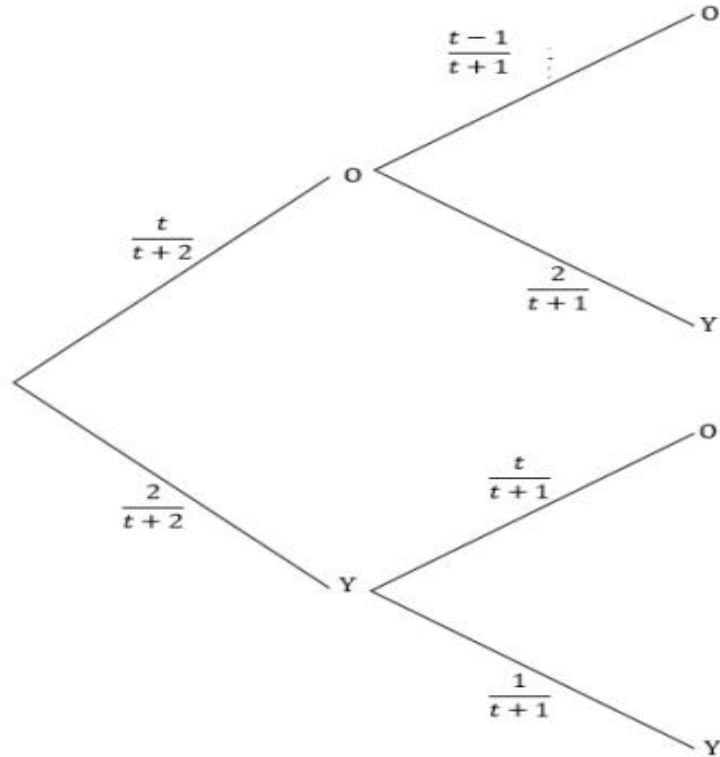
Activity 6

6.1	$\frac{8}{x} = 25\%$ $\frac{8}{x} = \frac{1}{4}$ $x = 32 \text{ balls}$
6.2	
6.3	$P(\text{same colour}) = P(R \cap R) + P(G \cap G) + P(Y \cap Y)$ $P(\text{same colour}) = \frac{7}{124} + \frac{15}{496} + \frac{153}{496}$ $P(\text{same colour}) = \frac{49}{124}$ $P(\text{same colour}) = 0,395$

Activity 7

7.1	
7.2	$P(B \cap B \cap B) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ $P(B \cap B \cap B) = \frac{1}{8}$
7.3	$P(\text{Alfred won 2 points}) = P(A \cap A \cap B) + P(A \cap B \cap A) + P(B \cap A \cap A)$ $P(\text{Alfred won 2 points}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ $P(\text{Alfred won 2 points}) = \frac{3}{8}$ $P(\text{Alfred won 2 points}) = 0,375$

Activity 8



$$P(\text{Same colour}) = P(O \cap O) + P(Y \cap Y)$$

$$\frac{52}{100} = \left(\frac{t}{t+2} \times \frac{t-1}{t+1} \right) + \left(\frac{2}{t+2} \times \frac{1}{t+1} \right)$$

$$\frac{52}{100} = \frac{t^2 - t + 2}{t^2 + 3t + 2}$$

$$52(t^2 + 3t + 2) = 100(t^2 - t + 2)$$

$$48t^2 - 256t + 96 = 0$$

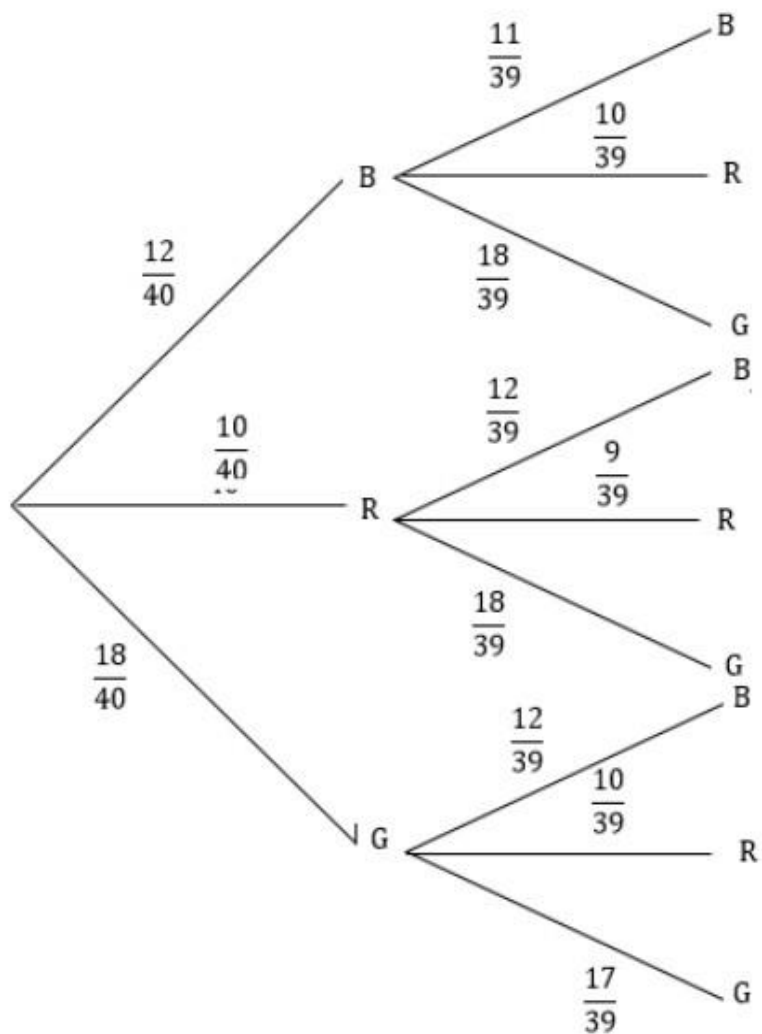
$$3t^2 - 16t + 2 = 0$$

$$t = \frac{16 \pm \sqrt{232}}{2 \times 3}$$

$$t = 0,12 \quad \text{or} \quad t = 5,21$$

$$\therefore t = 5$$

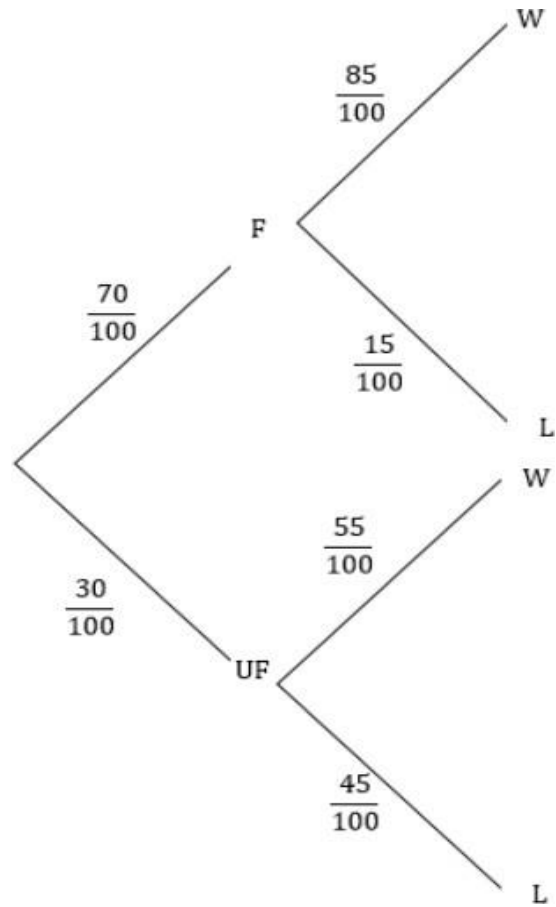
Activity 9



9.1 $P(G \cap G) = \frac{18}{40} \times \frac{17}{39} = 0,196$

9.2 $P(R \cap B) + P(B \cap R) = \left(\frac{10}{40} \times \frac{12}{39}\right) + \left(\frac{12}{40} \times \frac{10}{39}\right) = 0,154$

ACTIVITY 10



$$P(UF \text{ and } W) = \frac{30}{100} \times \frac{55}{100}$$

COUNTING PRINCIPLE SOLUTIONS

QUESTION 1

$$= 7^2 \sqrt{}$$

$$= 823\,543 \sqrt{}$$

$$= 7! \sqrt{}$$

$$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 5040 \sqrt{}$$

There are 3 vowels \Rightarrow 3 options for first position

There are 4 consonents \Rightarrow 4 options for last position

The remaining 5 letters can be arranged in $5 \times 4 \times 3 \times 2 \times 1$ ways

$$3 \times (5 \times 4 \times 3 \times 2 \times 1) \times 4 = 1440\sqrt{\sqrt{\sqrt{\sqrt{}}}}$$

QUESTION 2

$$= \frac{7!}{3! \times 2!} \sqrt{\sqrt{}}$$

$$= 420\sqrt{}$$

$$\text{number of ways} = \frac{6!}{3!} \sqrt{}$$

$$\text{number of ways} = 120\sqrt{}$$

QUESTION 3

$$= \frac{9!}{(9-5)!} \sqrt{\quad}$$

$$= 15\,120\sqrt{}$$

OR

$$= 9 \times 8 \times 7 \times 6 \times 5\sqrt{}$$

$$= 15\,120\sqrt{}$$

$$= 8 \times 7 \times 6 \times 5 \times 4\sqrt{v}$$

$$= 6\,720\sqrt{}$$

$$= 4^5 \sqrt{}$$

$$= 1\,024\sqrt{}$$

QUESTION 4

$$= 9 \times 8 \times 7 \times 6 \times 5\sqrt{}$$

$$= 15120\sqrt{}$$

4 consonants and 5 vowels

$$= 5 \times 5 \times 4 \times 3 \times 2 \times 1 \sqrt{\sqrt{}}$$

$$= 600 \sqrt{\sqrt{}}$$

OR

$$= 5 \times 5! \sqrt{\sqrt{}}$$

$$= 600 \sqrt{\sqrt{}}$$

QUESTION 5

$$= 10 \times 10 \times 10 \times 10 \times 10$$

$$= 10^5$$

$$= 100\,000$$

$$= 10 \times 9 \times 8 \times 7 \times 6 \sqrt{\sqrt{}}$$

$$= 30\,240 \sqrt{\sqrt{}}$$

OR

$$= \frac{10!}{5!} \sqrt{\sqrt{}}$$

$$= 30\,240 \sqrt{\sqrt{}}$$

Number of PINs that DO NOT contain 9s

$$= 9 \times 9 \times 9 \times 9 \times 9$$

$$= 59\,049 \sqrt{\sqrt{}}$$

$$P(\text{at least one } 9) = 1 - P(\text{no } 9\text{s}) \sqrt{\sqrt{}}$$

$$P(\text{at least one } 9) = 1 - \frac{59049}{100\,000} \sqrt{\sqrt{}}$$

$$P(\text{at least one } 9) = 0,41 \sqrt{\sqrt{}}$$

QUESTION 6

$= 14!$ $= 8,72 \times 10^{10} \sqrt{}$
$= 4! \times 5! \times 3! \times 4! \times 2! \sqrt{}$ $= 829\,440 \sqrt{}$
$= \frac{11!}{2! \times 2!} \sqrt{} \sqrt{}$ $= 9\,979\,200 \sqrt{}$
$= \frac{10!}{2! \times 2!} \sqrt{}$ $= 907\,200 \sqrt{}$
QUESTION 7
$= 7! \sqrt{}$ $= 5040 \sqrt{}$
$= 1 \times 5! \times 1 \sqrt{}$ $= 120 \sqrt{}$
$= 4! \times 4! \sqrt{} \sqrt{}$ $= 576 \sqrt{}$
$= 7 \times 6 \times 5 \times 4 \sqrt{}$ $= 840 \sqrt{}$
QUESTION 8
$= 26 \times 25 \times 24 \times 23 \times 22$ $= 7\,893\,600$ OR $= \frac{26!}{(26 - 5)!}$

$= \frac{26!}{21!}$ $= 7\,893\,600$
$= 24 \times 23 \times 22\sqrt{}$ $= 12\,144\sqrt{}$
$= 7!$ $= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1\sqrt{}$ $= 5\,040\sqrt{}$
$= 3! \times 5!\sqrt{}$ $= (3 \times 2 \times 1) \times (5 \times 4 \times 3 \times 2 \times 1)\sqrt{}$ $= 720\sqrt{}$

QUESTION 9
$= 5!\sqrt{}$ $= 5 \times 4 \times 3 \times 2 \times 1$ $= 120\sqrt{}$
$P(\text{having } S \text{ and } T \text{ as first two letters}) = \frac{2! \times 3!}{120}\sqrt{}$ $P(\text{having } S \text{ and } T \text{ as first two letters}) = \frac{2 \times 6}{120}$ $P(\text{having } S \text{ and } T \text{ as first two letters}) = \frac{1}{10}\sqrt{}$
QUESTION 10
$= 8!\sqrt{}$ $= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1\sqrt{}$

$= 40\,320\sqrt{}$
<p><i>girls can be seated together in 5!</i></p> <p><i>then girls and boys together can be seated in 4!</i></p> <p>$\therefore = 5! \times 4!$</p> <p>$= 120 \times 24$</p> <p>$= 2\,880$</p>
$P(\text{girls are together}) = \frac{5! \times 4!}{8!} \sqrt{}$ $P(\text{girls are together}) = \frac{2\,880}{40\,320}$ $P(\text{girls are together}) = 0,071\sqrt{}$
QUESTION 11
$= 7! \sqrt{}$ $= 5040\sqrt{}$
$= 4! \times 4! \sqrt{}$ $= 576\sqrt{}$
$P(\text{girls seated together}) = \frac{4! \times 4!}{7!} \sqrt{}$ $P(\text{girls seated together}) = \frac{576}{5040}$ $P(\text{girls seated together}) = \frac{4}{35} \sqrt{}$ $P(\text{girls seated together}) = 0,11\sqrt{} \quad [0,1142857143]$
$P(\text{girl, boy, girl, boy ...}) = \frac{4! \times 3!}{7!} \sqrt{}$ $P(\text{girl, boy, girl, boy ...}) = \frac{144}{5040} \sqrt{}$ $P(\text{girl, boy, girl, boy ...}) = \frac{1}{35}$ $P(\text{girl, boy, girl, boy ...}) = 0,03\sqrt{} \quad [0,02857142857]$

QUESTION 12

$$P(\text{girl} \dots \text{girl}) = \frac{7 \times 13! \times 6}{15!} \sqrt{\sqrt{}}$$

$$P(\text{girl} \dots \text{girl}) = \frac{1}{5} \sqrt{\sqrt{}}$$

$$P(\text{boy, girl, boy, girl} \dots) = \frac{8! \times 7!}{15!} \sqrt{\sqrt{}}$$

$$P(\text{boy, girl, boy, girl} \dots) = \frac{1}{6435} \sqrt{\sqrt{}}$$

QUESTION 13

$$= 6! \sqrt{\sqrt{}}$$

$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 720 \sqrt{\sqrt{}}$$

$$= 5! \times 2 \sqrt{\sqrt{}}$$

$$= 240 \sqrt{\sqrt{}}$$

Number of ways Mary is at an end of the row on the Left = $1 \times 5! \sqrt{\sqrt{}}$

Number of ways Mary is at an end of the row on the right = $5! \times 1 \sqrt{\sqrt{}}$

Total number of arrangements = $6!$

$$P(\text{Mary is at an end of the row}) = \frac{5! \times 1 + 1 \times 5!}{6!} \sqrt{\sqrt{}}$$

$$P(\text{Mary is at an end of the row}) = \frac{1}{3} \sqrt{\sqrt{}}$$

QUESTION 14

$$n(\text{codes}) = 20^4 \times 9^4 \sqrt{\sqrt{}}$$

$$n(\text{codes}) = 1\,049\,760\,000 \sqrt{\sqrt{}}$$

$n(\text{digits unequal even numbers}) = 20^4 \times 4!$ $n(\text{digits unequal even \#}) = 3\,840\,000$ $P(\text{digits unequal even \#}) = \frac{3\,840\,000}{1\,049\,760\,000} \sqrt{}$ $P(\text{digits unequal even \#}) = \frac{8}{2\,187} \sqrt{}$
QUESTION 15
$= 11! \sqrt{}$ $= 39\,916\,800 \sqrt{}$
Number of ways so that the shirts are together and trousers are together $= 7! \times 4! \times 2! \sqrt{} \sqrt{}$ $= 241\,920 \sqrt{}$
$P(\text{shirt at beginning and trouser at the end}) = \frac{7 \times 9! \times 4}{11!} \sqrt{} \sqrt{} \sqrt{}$ $P(\text{shirt at the beginning and trouser at the end}) = \frac{14}{55} \sqrt{}$
QUESTION 16
$= 7! \sqrt{}$ $= 5040 \sqrt{}$
$\text{koketso and marvin can sit together in } 2! \, 6! \sqrt{}$ $\text{they will not sit together in } 7! - 2! \, 6! \text{ ways} \sqrt{}$ $\therefore = 7! - 2! \, 6! \sqrt{}$ $= 3600 \sqrt{}$
$P(\text{sit at 6th position}) = \frac{1}{6} \sqrt{} \sqrt{}$